Uncertainty Approaches for Solving Generalized Machine Maintenance Problem

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Abstract—**In this paper, the generalized machine maintenance problem is formulated as linear programming model. The objective is to maximize the percentage production hours available per maintenance cycle of each machine. Data in many real life engineering and economic problems suffers from inexactness. There are different approaches to deal with uncertain optimization problems. In this paper two different approaches of uncertainty, Fuzzy programming and rough interval programming approaches will be introduced. We deal the concerned problem with uncertain data in coefficients of the constraints for the two approaches. A numerical example is introduced to clarify the two proposed approaches. A comparison study between the obtained results of the two proposed approaches and the results of interval approach for Samir A. and Marwa Sh [1] will be introduced.**

Index Terms—**Machine Maintenance Problem, Fuzzy Programming, Rough Interval Programming, Linear Programming.**

I. INTRODUCTION

Today's the goal of a business is to make money now and in the future. Every organization within the business should proceed to accomplish that goal. The most important part for achieving this goal is keeping maintenance. Maintenance managers have begun to get roles as more than fix machines. In the past, the machine maintenance role is to serve the needs of factory equipment users by waiting for a machine to break and then repairing it. Now waiting for maintenance has no place in a world-class company. So, today's maintenance departments must team with production departments to ensure that machinery is kept running and get optimum production with no losing time.

Allan et-al [2] defined maintenance as all activity in keeping equipment in working order and pointed out that the amount and type of maintenance applied depends strongly on its cost and safety implication of the system failure. Joseph [3] reported that maintenance philosophy plays an important role in determining overall availability. Kareem and Aderoba [4] presented a case study for effective maintenance and manpower planning strategy. Samir A. and Marwa Sh. [1] introduced generalized machine maintenance problem under uncertainty.

In optimizing conventional problems, all of the coefficients are precise numbers. However, in the real world systems, the coefficients contained in the objective and constraints are imprecise. Galbraith [5] defines uncertainty as the difference between the amount of information required to perform a task and the amount of information already possessed. There are many approaches that can be adopted to treat uncertain systems. Two of these approaches are fuzzy approach and rough interval approach. Fuzzy optimization problems were considered first by Bellman and Zadeh [6]. Thereafter, Tanaka et al. [7] developed the concept of fuzzy mathematical programming in a general level. In fuzzy programming the constraints and/or objective function are viewed as fuzzy sets and their membership functions also need to be known. The first formulation of fuzzy linear programming is proposed by Zimmermann [8]. Ebrahimnejad et al. [9] proposed bounded linear programs with trapezoidal fuzzy numbers. Maleki et al. [10] introduced linear programming with fuzzy variables.

The idea of the other approach, rough interval, was proposed by Pawlak [11] as a new mathematical tool to deal with vague concepts. The point of strength of rough set theory than other methods is that, it requires no additional information, external parameters, models or functions to determine membership function and probability distributions. It only uses the information presented within the given data as presented in Düntsch and Gediga [12]. Rough set is adopted in many problems as Krysinski proposed in pharmacology [13], Weiguo et al proposed in decision algorithms [14] and Arabani and Nashaei proposed in civil engineering [15]. Osman et al. [16] calssified the rough programming problems into three classes according to the place of the roughness. Youness [17] introduced a rough optimal solution and a rough saddle point. Hamzehee et al [18] introduced a problem with rough interval coefficients for linear programming.

In our concerned study, we treat one problem of factory machine maintenance. Our objective is to maximize the percentage production hours available per maintenance cycle of each machine. We introduce the uncertain optimization problems using rough set analysis besides to fuzzy programming approach. We consider that each of the maximum spare part cost, depreciation cost and the maximum waiting time of each machine to be uncertainty rather than constant.

The rest of this paper is organized as follows. The problem formulation is described in section II. In section III the machine maintenance problem with fuzzy is proposed. In section IV the same problem is proposed using rough interval as uncertainty in constraints. A numerical example is provided in section V to clarify the proposed approaches. In section VI a comparison between the obtained results of the two proposed approaches and the results of interval

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approach [1] is introduced. Finally, the conclusions are presented in section VII.

II. Problem Formulation

We have a machine maintenance problem to maximize the percentage production hours available per maintenance cycle. First we present the used notions

Parameters

p_i : Production hours available per maintenance cycle of machine i where $i = 1,\ldots,n$.

mc_i : Manpower cost per hour to machine i .

sc_i : Spare part cost per hour to machine i .

dc_i : Depreciation cost per hour to machine i .

Wt_i : The maximum waiting time of machine i .

MC : The maximum manpower cost.

SC : The maximum spare part cost.

DC : The maximum depreciation cost.

t_i :Waiting time of each machine i.

n : number of machines.

Decision variables

x_i : The number of crew allocated to machine i .

The mathematical model:

We exhibit the following machine maintenance problem [1]

$$\max Z = \sum_{i=1}^{n} p_i x_i \tag{1}$$

subject to

$$\sum_{i=1}^{n} mc_i x_i \le MC \tag{2}$$

$$\sum_{i=1}^{n} sc_i x_i \le SC \tag{3}$$

$$\sum_{i=1}^{n} dc_i x_i \le DC \tag{4}$$

$$t_i x_i \le Wt_i \qquad i = 1,\ldots,n \tag{5}$$

$$x_i \ge 0 \qquad i = 1,\ldots,n \tag{6}$$

The objective function (1) maximizes the percentage production hours available per maintenance cycle of each machine. Constraint (2) is on manpower cost associated with the maintenance of each machine. Constraint (3) is on spare part cost associated with the maintenance of each machine. Constraint (4) is on depreciation cost associated with the maintenance of each machine. Constraint (5) is on the maximum hour available for maintenance in each maintenance cycle.

In what follows we consider the parameters that represent each of the maximum spare part cost (SC), The maximum depreciation cost (DC) and the maximum waiting time of machine i (Wt_i) are uncertainty parameters.

III. The Machine Maintenance Problem with Fuzzy Parameters

The optimization model of machine maintenance problem with fuzzy parameters is formulated as follows:

$$\max Z = \sum_{i=1}^{n} p_i x_i \tag{7}$$

subject to

$$\sum_{i=1}^{n} mc_i x_i \le MC \tag{8}$$

$$\sum_{i=1}^{n} sc_i x_i \le SC \tag{9}$$

$$\sum_{i=1}^{n} dc_i x_i \le DC \tag{10}$$

$$t_i x_i \le Wt_i \qquad i = 1,\ldots,n \tag{11}$$

$$x_i \ge 0 \qquad i = 1,\ldots,n \tag{12}$$

Where SC , DC and Wt_i represent fuzzy parameters involved in the constraints with their membership functions are μ_{SC} , μ_{DC} and μ_{wt_i} respectively.

Treatment procedure

Therefore problem (7)-(12) can be understood as the following non fuzzy problem:

$$\max Z = \sum_{i=1}^{n} p_i x_i \tag{13}$$

subject to

$$\sum_{i=1}^{n} mc_i x_i \le MC \tag{14}$$

$$\sum_{i=1}^{n} sc_i x_i \le SC \tag{15}$$

$$\sum_{i=1}^{n} dc_i x_i \le DC \tag{16}$$

$$t_i x_i \le Wt_i \qquad i = 1,\ldots,n \tag{17}$$

$$SC \in L_\alpha(SC) \tag{18}$$

$$DC \in L_\alpha(DC) \tag{19}$$

$$Wt_i \in L_\alpha(Wt_i) \tag{20}$$

$$x_i \ge 0 \qquad i = 1,\ldots,n \tag{21}$$

$L_\alpha(SC)$, $L_\alpha(DC)$ and $L_\alpha(Wt_i)$ are the α-level set of the fuzzy numbers SC, DC and Wt_i respectively, where $L_\alpha(SC) = [SC_1 + \alpha(SC_2 - SC_1),\ SC_3 - \alpha(SC_3 - SC_2)]$, with triangular fuzzy numbers $SC = (SC_1, SC_2, SC_3)$, $L_\alpha(DC) = [DC_1 + \alpha(DC_2 - DC_1),\ DC_3 - \alpha(DC_3 - DC_2)]$ with triangular fuzzy numbers $DC = (DC_1, DC_2, DC_3)$ and $L_\alpha(Wt_i) = [Wt_{i1} + \alpha(Wt_{i2} - Wt_{i1}),\ Wt_{i3} - \alpha(Wt_{i3} - Wt_{i2})]$ with triangular fuzzy numbers $Wt_i = (Wt_{i1}, Wt_{i2}, Wt_{i3})$ such that $\alpha \in [0,1]$. Problem (13)-(21) can be written in the following equivalent form:

$$\max Z = \sum_{i=1}^{n} p_i x_i \tag{22}$$

subject to

$$\sum_{i=1}^{n} mc_i x_i \le MC \tag{23}$$

$$\sum_{i=1}^{n} sc_i x_i \le SC \tag{24}$$

$$\sum_{i=1}^{n} dc_i x_i \le DC \tag{25}$$

$$t_i x_i \le Wt_i \qquad i = 1, \dots, n \tag{26}$$

$$S1 \le SC \le S2 \tag{27}$$

$$D1 \le DC \le D2 \tag{28}$$

$$Wt2_i \le Wt_i \le Wt1_i \tag{29}$$

$$x_i \ge 0 \qquad i = 1, \dots, n \tag{30}$$

where $S1, S2, D1, D2$ and $Wt1_i$ and $Wt2_i$ are lower and upper bounds on SC, DC and Wt_i respectively and they are constants. It is should be noted that the set of constraints (18-20) have been replaced by the set of constraints (27-29).

IV. The Machine Maintenance Problem with Rough Parameters

The optimization model of machine maintenance problem with rough interval parameters is formulated as follows:

$$\max Z = \sum_{i=1}^{n} p_i x_i \tag{31}$$

subject to

$$\sum_{i=1}^{n} mc_i x_i \le MC \tag{32}$$

$$\sum_{i=1}^{n} sc_i x_i \le SC \tag{33}$$

$$\sum_{i=1}^{n} dc_i x_i \le DC \tag{34}$$

$$t_i x_i \le Wt_i \qquad i = 1, \dots, n \tag{35}$$

$$x_i \ge 0 \qquad i = 1, \dots, n \tag{36}$$

Where SC, DC and Wt_i are rough interval parameters such that:

$$SC = \left(\left[\underline{SC^L}, \underline{SC^U}\right], \left[\overline{SC^L}, \overline{SC^U}\right]\right)$$

$$DC = \left(\left[\underline{DC^L}, \underline{DC^U}\right], \left[\overline{DC^L}, \overline{DC^U}\right]\right)$$

$$Wt_i = \left(\left[\underline{Wt_i^L}, \underline{Wt_i^U}\right], \left[\overline{Wt_i^L}, \overline{Wt_i^U}\right]\right)$$

Therefore problem (31)-(36) can be written as

$$\max Z = \sum_{i=1}^{n} p_i x_i \tag{37}$$

subject to

$$\sum_{i=1}^{n} mc_i x_i \le MC \tag{38}$$

$$\sum_{i=1}^{n} sc_i x_i \le \left(\left[\underline{SC^L}, \underline{SC^U}\right], \left[\overline{SC^L}, \overline{SC^U}\right]\right) \tag{39}$$

$$\sum_{i=1}^{n} dc_i x_i \le \left(\left[\underline{DC^L}, \underline{DC^U}\right], \left[\overline{DC^L}, \overline{DC^U}\right]\right) \tag{40}$$

$$t_i x_i \le \left(\left[\underline{Wt_i^L}, \underline{Wt_i^U}\right], \left[\overline{Wt_i^L}, \overline{Wt_i^U}\right]\right) \qquad i = 1, \dots, n \tag{41}$$

$$x_i \ge 0 \qquad i = 1, \dots, n \tag{42}$$

A. Treatment procedure

We will treat the uncertainty represented by rough interval coefficient in the constraints as follows:

First we will divide the model for the machine maintenance problem with random rough coefficients in constraints into lower and upper interval problems.

B. Lower interval problem:

$$\max Z = \sum_{i=1}^{n} p_i x_i \tag{43}$$

subject to

$$\sum_{i=1}^{n} mc_i x_i \le MC \tag{44}$$

$$\sum_{i=1}^{n} sc_i x_i \le \left[\underline{SC}^L, \underline{SC}^U \right] \quad (45)$$

$$\sum_{i=1}^{n} dc_i x_i \le \left[\underline{DC}^L, \underline{DC}^U \right] \quad (46)$$

$$t_i x_i \le \left[\underline{Wt_i}^L, \underline{Wt_i}^U \right] \qquad i = 1, \ldots, n \quad (47)$$

$$x_i \ge 0 \qquad i = 1, \ldots, n \quad (48)$$

Upper interval problem

$$\max Z = \sum_{i=1}^{n} p_i x_i \quad (49)$$

subject to

$$\sum_{i=1}^{n} mc_i x_i \le MC \quad (50)$$

$$\sum_{i=1}^{n} sc_i x_i \le \left[\overline{SC}^L, \overline{SC}^U \right] \quad (51)$$

$$\sum_{i=1}^{n} dc_i x_i \le \left[\overline{DC}^L, \overline{DC}^U \right] \quad (52)$$

$$t_i x_i \le \left[\overline{Wt_i}^L, \overline{Wt_i}^U \right] \qquad i = 1, \ldots, n \quad (53)$$

$$x_i \ge 0 \qquad i = 1, \ldots, n \quad (54)$$

Then each of the lower and upper interval problems will be divided into two deterministic problems. So the lower interval problem (43)-(48) will give the following two linear problems $\mathbf{P_1}$ and $\mathbf{P_2}$:

$\mathbf{P_1}$

$$\max Z = \sum_{i=1}^{n} p_i x_i \quad (55)$$

subject to

$$\sum_{i=1}^{n} mc_i x_i \le MC \quad (56)$$

$$\sum_{i=1}^{n} sc_i x_i \le \underline{SC}^L \quad (57)$$

$$\sum_{i=1}^{n} dc_i x_i \le \underline{DC}^L \quad (58)$$

$$t_i x_i \le \underline{Wt_i}^L \qquad i = 1, \ldots, n \quad (59)$$

$$x_i \ge 0 \qquad i = 1, \ldots, n \quad (60)$$

P_2

$$\max Z = \sum_{i=1}^{n} p_i x_i \quad (61)$$

subject to

$$\sum_{i=1}^{n} mc_i x_i \le MC \quad (62)$$

$$\sum_{i=1}^{n} sc_i x_i \le \underline{SC}^U \quad (63)$$

$$\sum_{i=1}^{n} dc_i x_i \le \underline{DC}^U \quad (64)$$

$$t_i x_i \le \underline{Wt_i}^U \qquad i = 1, \ldots, n \quad (65)$$

$$x_i \ge 0 \qquad i = 1, \ldots, n \quad (66)$$

Where, the solutions of problems P_1 and P_2 give the surly optimal range of our machine maintenance problem with rough interval (37-42).

Similarly the upper interval problem (49)-(54) will divided into the following two linear problems $\mathbf{P_3}$ and $\mathbf{P_4}$:

$\mathbf{P_3}$

$$\max Z = \sum_{i=1}^{n} p_i x_i \quad (67)$$

subject to

$$\sum_{i=1}^{n} mc_i x_i \le MC \quad (68)$$

$$\sum_{i=1}^{n} sc_i x_i \le \overline{SC}^L \quad (69)$$

$$\sum_{i=1}^{n} dc_i x_i \le \overline{DC}^L \quad (70)$$

$$t_i x_i \le \overline{Wt_i}^L \qquad i = 1, \ldots, n \quad (71)$$

$$x_i \ge 0 \qquad i = 1, \ldots, n \quad (72)$$

$\mathbf{P_4}$

$$\max Z = \sum_{i=1}^{n} p_i x_i \quad (73)$$

subject to

$$\sum_{i=1}^{n} mc_i x_i \le MC \quad (74)$$

$$\sum_{i=1}^{n} sc_i x_i \le \overline{SC}^U \quad (75)$$

$$\sum_{i=1}^{n} dc_i x_i \le \overline{DC}^U \quad (76)$$

$$t_i x_i \le \overline{Wt_i}^U \qquad i = 1, \ldots, n \quad (77)$$

$$x_i \geq 0 \qquad i = 1, \dots, n \tag{78}$$

Where, the solutions of problems P_3 and P_4 give the possibly optimal range of our machine maintenance problem with rough interval (37-42).

V. Numerical Example

According to the data of an example which is reported in [1] consider the instance of machine maintenance problem to maximize the percentage production hours available per maintenance cycle of each machine is given by Table I, II and III.

TABLE I: Deterministic Data of Machine Maintenance Problem

Manpower cost /hr. mci ($)	Sparepart cost / hr. sci ($)	Deprecation cost/hr. dci ($)	Production hours p_i	Waiting time t_i
14.02	21.8	1934.52	0.998hr.	$t_1 = 0.3$
28.04	43.6	644.84	0.998hr.	$t_2 = 0.9$
35.05	45.5	322.42	0.999hr.	$t_3 = 1.5$
46.69	72.6	806.05	0.998hr.	$t_4 = 1$
70.1	109	806.05	0.997hr.	$t_5 = 1.8$
84.2	130.9	1934.52	0.994hr.	$t_6 = 2.9$
112.16	174	806.05	0.996hr.	$t_7 = 0.9$
14.02	21.8	967.26	0.999hr.	$t_8 = 0.5$
42.06	65.5	806.05	0.998hr.	$t_9 = 1.2$

TABLE II: Data for Fuzzy Approach

The maximum spare part cost	$SC = (600, 650, 700)$	$\alpha_{SC} = 0.5$
The maximum depreciation cost	$DC = (8000, 8500, 9000)$	$\alpha_{DC} = 0.6$
The maximum waiting time of machine 1	$Wt_1 = (0.5, 1, 1.5)$	$\alpha_1 = 0.4$
The maximum waiting time of machine 2	$Wt_2 = (1, 1.5, 2)$	$\alpha_2 = 0.9$
The maximum waiting time of machine 3	$Wt_3 = (2, 2.5, 3)$	$\alpha_3 = 0.3$
The maximum waiting time of machine 4	$Wt_4 = (1, 2, 3)$	$\alpha_4 = 0.7$
The maximum waiting time of machine 5	$Wt_5 = (1, 2.5, 5)$	$\alpha_5 = 0.8$
The maximum waiting time of machine 6	$Wt_6 = (2, 4, 6)$	$\alpha_6 = 0.9$
The maximum waiting time of machine 7	$Wt_7 = (1, 3, 4)$	$\alpha_7 = 0.1$
The maximum waiting time of machine 8	$Wt_8 = (1, 1.5, 2)$	$\alpha_8 = 0.5$
The maximum waiting time of machine 9	$Wt_9 = (1, 2, 3)$	$\alpha_9 = 0.9$

By substitute in the model (7) - (12) and using the optimization approach which is described in section III, the deterministic form for this example is obtained as follows:

$$\max Z = 0.998x_1 + 0.998x_2 + 0.999x_3 + 0.998x_4 + 0.997x_5 + 0.994x_6 + 0.996x_7 + 0.999x_8 + 0.998x_9$$

subject to

$$14.02x_1 + 28.04x_2 + 35.05x_3 + 46.69x_4 + 70.1x_5 + 84.2x_6 + 112.16x_7 + 14.02x_8 + 42.06x_9 \leq 446.1$$

$$21.8x_1 + 43.6x_2 + 54.5x_3 + 72.6x_4 + 109x_5 + 130.9x_6 + 174x_7 + 21.8x_8 + 65.56x_9 - x_{10} \leq 0$$

$$1934x_1 + 644.84x_2 + 322.42x_3 + 806.05x_4 + 806.05x_5 + 1934.52x_6 + 806.05x_7 + 967.26x_8 + 806.05x_9 - x_{11} \leq 0$$

$$0.3x_1 - x_{12} \leq 0 \quad 0.9x_2 - x_{13} \leq 0 \quad 1.5x_3 - x_{14} \leq 0$$
$$x_4 - x_{15} \leq 0 \quad 1.8x_5 - x_{16} \leq 0 \quad 2.9x_6 - x_{17} \leq 0$$
$$0.9x_7 - x_{18} \leq 0 \quad 0.5x_8 - x_{19} \leq 0 \quad 1.2x_9 - x_{20} \leq 0$$
$$x_{10} \leq 675 \quad x_{10} \geq 625 \quad x_{11} \leq 8700 \quad x_{11} \geq 8300$$
$$x_{12} \leq 1.3 \quad x_{12} \geq 0.7 \quad x_{13} \leq 1.55 \quad x_{13} \geq 1.45$$
$$x_{14} \leq 2.85 \quad x_{14} \geq 2.15 \quad x_{15} \leq 2.3 \quad x_{15} \geq 1.7$$
$$x_{16} \leq 3 \quad x_{16} \geq 2.2 \quad x_{17} \leq 4.2 \quad x_{17} \geq 3.8$$
$$x_{18} \leq 3.9 \quad x_{18} \geq 1.2 \quad x_{19} \leq 1.75 \quad x_{19} \geq 1.25$$
$$x_{20} \leq 2.1 \quad x_{20} \geq 1.9$$

$$x_i \geq 0 \qquad \forall i$$

Then we get the following solution:

$$x_1 = x_6 = x_7 = 0, x_2 = 1.7222, x_3 = 1.9, x_4 = 2.3, x_5 = 1.4432, x_8 = 2.6354, x_9 = 1.75, x_{10} = 675, x_{11} = 8700, x_{12} = 0.7, x_{13} = 1.55, x_{14} = 2.85, x_{15} = 2.3, x_{16} = 2.5977, x_{17} = 3.8, x_{18} = 1.2, x_{19} = 1.75, x_{20} = 2.1$$

labors and objective function value = 11.7303 hours.

TABLE III: Data for Rough Interval Approach

The maximum spare part cost	$SC = ([600, 650], [570, 700])$
The maximum depreciation cost	$DC = ([8000, 8500], [7800, 9000])$
The maximum waiting time of machine1	$Wt_1 = ([0.5, 1], [0.3, 1.5])$
The maximum waiting time of machine2	$Wt_2 = ([1, 1.5], [0.8, 2])$
The maximum waiting time of machine3	$Wt_3 = ([2, 2.5], [1.8, 3])$
The maximum waiting time of machine4	$Wt_4 = ([1, 2], [0.7, 3])$
The maximum waiting time of machine5	$Wt_5 = ([1, 2.5], [0.5, 5])$
The maximum waiting time of machine6	$Wt_6 = ([2, 4], [1, 6])$
The maximum waiting time of machine7	$Wt_7 = ([1, 3], [1, 4])$
The maximum waiting time of machine8	$Wt_8 = ([1, 1.5], [0.6, 2])$
The maximum waiting time of machine9	$Wt_9 = ([1, 2], [0.5, 3])$

By substitute in the models (55) - (60), (61) - (66), (67) - (72) and (73) - (78) by using the optimization approach which is described in section IV, the deterministic problems for this example is obtained as follows:

P1

$$\max Z = 0.998x_1 + 0.998x_2 + 0.999x_3 + 0.998x_4 + 0.997x_5 + 0.994x_6 + 0.996x_7 + 0.999x_8 + 0.998x_9$$

subject to

$$14.02x_1 + 28.04x_2 + 35.05x_3 + 46.69x_4 + 70.1x_5 + 84.2x_6 + 112.16x_7 + 14.02x_8 + 42.06x_9 \le 446.1$$

$$21.8x_1 + 43.6x_2 + 54.5x_3 + 72.6x_4 + 109x_5 + 130.9x_6 + 174x_7 + 21.8x_8 + 65.56x_9 \le 600$$

$$1934x_1 + 644.84x_2 + 322.42x_3 + 806.05x_4 + 806.05x_5 + 1934.52x_6 + 806.05x_7 + 967.26x_8 + 806.05x_9 \le 8000$$

$$0.3x_1 \le 0.5,\ 0.9x_2 \le 1,\ 1.5x_3 \le 2,\ x_4 \le 1,\ 1.8x_5 \le 1,\ 2.9x_6 \le 2,\ 0.9x_7 \le 1,\ 0.5x_8 \le 1,\ 1.2x_9 \le 1$$

$$x_i \ge 0 \qquad \forall i$$

Whose solution is

$$x_1 = 1.0845,, x_2 = x_7 = 01.1111, x_3 = 1.3333, x_4 = 1, x_5 = 0.5556, x_6 = 0, x_8 = 2, x_9 = 0.8331$$

labors and objective function value = 9.0114 hours.

P2

$$\max Z = 0.998x_1 + 0.998x_2 + 0.999x_3 + 0.998x_4 + 0.997x_5 + 0.994x_6 + 0.996x_7 + 0.999x_8 + 0.998x_9$$

subject to

$$14.02x_1 + 28.04x_2 + 35.05x_3 + 46.69x_4 + 70.1x_5 + 84.2x_6 + 112.16x_7 + 14.02x_8 + 42.06x_9 \le 446.1$$

$$21.8x_1 + 43.6x_2 + 54.5x_3 + 72.6x_4 + 109x_5 + 130.9x_6 + 174x_7 + 21.8x_8 + 65.56x_9 \le 650$$

$$1934x_1 + 644.84x_2 + 322.42x_3 + 806.05x_4 + 806.05x_5 + 1934.52x_6 + 806.05x_7 + 967.26x_8 + 806.05x_9 \le 8500$$

$$0.3x_1 \le 1,\ 0.9x_2 \le 1.5,\ 1.5x_3 \le 2.5,\ x_4 \le 2,\ 1.8x_5 \le 2.5,\ 2.9x_6 \le 4,\ 0.9x_7 \le 3,\ 0.5x_8 \le 1.5,\ 1.2x_9 \le 2$$

$$x_i \ge 0 \qquad \forall i$$

Whose solution is

$$x_1 = x_6 = 0, x_2 = x_3 = x_9 = 1.6667, x_4 = 2, x_5 = 1.3889, x_7 = 0.1103, x_8 = 2.8162$$

labors and objective function value =11.2956 hours.

The solutions of P1 and P2 give us the surly optimal range of the given example

P3

$$\max Z = 0.998x_1 + 0.998x_2 + 0.999x_3 + 0.998x_4 + 0.997x_5 + 0.994x_6 + 0.996x_7 + 0.999x_8 + 0.998x_9$$

subject to

$$14.02x_1 + 28.04x_2 + 35.05x_3 + 46.69x_4 + 70.1x_5 + 84.2x_6 + 112.16x_7 + 14.02x_8 + 42.06x_9 \le 446.1$$

$$21.8x_1 + 43.6x_2 + 54.5x_3 + 72.6x_4 + 109x_5 + 130.9x_6 + 174x_7 + 21.8x_8 + 65.56x_9 \le 570$$

$$1934x_1 + 644.84x_2 + 322.42x_3 + 806.05x_4 + 806.05x_5 + 1934.52x_6 + 806.05x_7 + 967.26x_8 + 806.05x_9 \le 7800$$

$$0.3x_1 \le 0.3,\ 0.9x_2 \le 0.8,\ 1.5x_3 \le 1.8,\ x_4 \le 0.7,\ 1.8x_5 \le 0.5,\ 2.9x_6 \le 1,\ 0.9x_7 \le 1,\ 0.5x_8 \le 0.6,\ 1.2x_9 \le 0.5$$

$$x_i \ge 0 \qquad \forall i$$

Whose solution is

$$x_1 = 1, x_2 = 0.8889, x_3 = 1.2, x_4 = 0.7, x_5 = 0.2778, x_6 = 0.3448, x_7 = 1.1111, x_8 = 1.2, x_9 = 0.416$$

labors and objective function value = 7.1235 hours.

P4

$$\max Z = 0.998x_1 + 0.998x_2 + 0.999x_3 + 0.998x_4 + 0.997x_5 + 0.994x_6 + 0.996x_7 + 0.999x_8 + 0.998x_9$$

subject to

$$14.02x_1 + 28.04x_2 + 35.05x_3 + 46.69x_4 + 70.1x_5 + 84.2x_6 + 112.16x_7 + 14.02x_8 + 42.06x_9 \le 446.1$$

$$21.8x_1 + 43.6x_2 + 54.5x_3 + 72.6x_4 + 109x_5 + 130.9x_6 + 174x_7 + 21.8x_8 + 65.56x_9 \le 700$$

$$1934x_1 + 644.84x_2 + 322.42x_3 + 806.05x_4 + 806.05x_5 + 1934.52x_6 + 806.05x_7 + 967.26x_8 + 806.05x_9 \le 9000$$

$$0.3x_1 \le 1.5,\ 0.9x_2 \le 2,\ 1.5x_3 \le 3,\ x_4 \le 3,\ 1.8x_5 \le 5,\ 2.9x_6 \le 6,\ 0.9x_7 \le 4,\ 0.5x_8 \le 2,\ 1.2x_9 \le 3$$

$$x_i \geq 0 \qquad \forall i$$

Whose solution is

$$x_1 = x_6 = x_7 = 0, x_2 = 2.222, x_3 = 2, x_4 = 3,$$
$$x_5 = 0.5545, x_8 = 2.1111, x_9 = 2.5$$

labors and objective function value = 12.3666 hours.

The solutions of P_3 and P_4 give us the possibly optimal range of the given example. So the interval [9.0114,11.2956] is the surly optimal range, the interval [7.1235, 12.3666] is the possibly optimal range and ([9.0114, 11.2956], [7.1235, 12.3666]) is the rough optimal range. The solutions of P_1 and P_2 give the completely satisfactory solutions and the solutions of P_3 and P_4 give the rather satisfactory solutions.

VI. Comparison Study

We test the efficiency of three uncertainty approaches for generalized machine maintenance problem. The three approaches are rough approach, fuzzy approach and interval approach. For the first two suggested approaches we assumed the parameters that represent each of the maximum spare part cost (SC), the maximum depreciation cost (DC) and the maximum waiting time of machine i (Wt_i) are uncertainty parameters. While in model of Samir A. and Marwa Sh. [1] the production hours (p_i) and the maximum waiting time of machine i (Wt_i) are uncertainty parameters.

For rough problem we obtained four deterministic problems and so we have four solutions, two of them give the completely satisfactory solutions and the others give the rather satisfactory solutions. Fuzzy approach and interval approach [1] have only one deterministic problem and gives one solution. Table IV clarifies the obtained results for the three approaches.

TABLE IV: Results of the Three Uncertainty Approaches

Points of comparison	The obtained results by rough approach	The obtained results by fuzzy approach	The obtained results by interval approach [1]
number of machine n	n= 9	n= 9	n= 9
Objective function values	P_1 and P_2 problems give the surly objective function optimal range which is [9.0114, 112956.] hours P_3 and P_4 problems give the possibly objective function optimal range which is [7.1235, 12.3666] hours	Objective function value =11.7303 hours	Objective function value =11.98 hours
Variables	for P_1 problem $x_1 = 1.0845$, $x_2 = x_7 = 01.1111$, $x_3 = 1.3333$, $x_4 = 1$, $x_5 = 0.5556$, $x_6 = 0$, $x_8 = 2$, $x_9 = 0.8331$ labors for P_2 problem $x_1 = x_6 = 0$, $x_2 = x_3 =$ $x_9 = 1.6667$, $x_4 = 2$, $x_5 = 1.3889$, $x_7 = 0.1103$, $x_8 = 2.8162$ for P_3 problem $x_5 = 0.2778$, $x_6 = 0.3448$, $x_7 = 1.1111$, $x_8 = 1.2$, $x_9 = 0.416$ labors for P_4 problem $x_1 = x_6 = x_7 = 0$, $x_2 = 2.222$, $x_3 = 2, x_4 = 3$, $x_5 = 0.5545$, $x_8 = 2.1111$, $x_9 = 2.5$ labors	$x_1 = x_6 = x_7 = 0$, $x_2 = 1.7222$, $x_3 = 1.9$, $x_4 = 2.3$, $x_5 = 1.4432$, $x_8 = 2.6354$, $x_9 = 1.75$ labors and $x_{10} = 675$, $x_{11} = 8700$, $x_{12} = 0.7$, $x_{13} = 1.55$, $x_{14} = 2.85$, $x_{15} = 2.3$, $x_{16} = 2.5977$, $x_{17} = 3.8$, $x_{18} = 1.2$, $x_{19} = 1.75$, $x_{20} = 2.1$ $x_1 = 1$, $x_2 = 0.8889$, $x_3 = 1.2$, $x_4 = 0.7$,	$x_1 = x_2 =$ $x_5 = x_8 = 1$, $x_3 = x_4 = 2$, $x_6 = x_7 = 0$, $x_9 = 3$ labors

From the obtained results in Table IV, it is clear that the objective function's values are very near to each other for the three uncertainty approaches. The rough approach is preferred because the four solutions obtained by this approach allow a wide range for decision maker to choose a suitable one.

VII. CONCLUSION

In this paper, we discussed the generalized machine maintenance problem. In our work two different approaches of uncertainty, Fuzzy programming and rough interval programming approaches are applied. The uncertain data has been presented in coefficients of the constraints for two approaches. We enhanced the percentage production hours available per maintenance cycle of each machine in the two approaches. A numerical example is introduced to clarify the two proposed approaches. A comparison study was proposed between fuzzy, rough and interval approach to clarify the most efficient method of them.

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