

Motion Prediction of Underwater Sensors

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Abstract — In this work, we will simulate the motion of a single underwater sensor knowing the current velocity to predict its location and velocity during certain time frame using a numerical approach of non-linear time-dependent partial differential equations and develop numerical computer programming code to solve the equations.

The underwater sensor are used to collect data for many scientific and practical reasons all the sensor collected data without specifying the sensor location and time will be missing lowers valuable information and by simulating the sensor motion numerically will have many values and impact on the underwater sensor industries as this will lead to less power consumption sensors with smaller size and less network coverage required.

This paper will study the kinetics of the underwater sensor which will resulted to a set of non-linear time-dependent partial differential equations that can be solved analytically and computer programming simulation is developed to solve the equations and predict the motion of underwater sensor.

Different scenarios considered in the work such as simulating the result for different sensor's density and the effect on its final position. Also, the result will include the sensor velocity simulation and comparison with the sea current velocity.

This work is limited to the motion prediction of single underwater sensor and the result is only for mechanical aspect of the problem, the networks connectivity or coverage is out-of-scope.

Index Terms — Analytical Solution, Underwater Sensor, Motion Prediction.

I. INTRODUCTION

Underwater sensors are widely used these days for many applications related to under-water world. The needs and usage of underwater sensors are more important every day due to the sensitivity of their usage and the applications that required underwater sensors. They are used in environmental sciences to sample and collect mainly in pollution monitoring, tsunami warning, or underwater life analysis. They are also important for harbor or military surveillance [1], [2].

Too many factors are involved for the accuracy and quality of sensors reading and data collected such as the WIFI network coverage, motors, battery-life and the sensor location. Manly GPS systems are used to detect the sensor exact location and usually these systems are power consuming, and it is preferable to minimize the number of GPS location reading for power rationalization and longer sensor lifetime [2].

One of the main tasks of underwater sensor is localization, since all other information without specifying the sensor

location and time will be missing data and not much valuable information [3].

The objective of this study is to develop an analytical model for the mobility of an underwater sensor. This will mainly lead to the establishment of a set of non-linear time-dependent partial differential equations. Then, use numerical computing software to solve the resulted set of partial differential equations using non-linear algorithms based either on implicit or explicit schemes.

Due to the importance, the increasing use, the challenges may be faced, and the sensitivity of data collected by underwater sensors due to its application has resulted to high demand in studies. Most studies in this field investigated the subject of network connection and communication and sensors distribution for better area coverage, also many studies talked about the underwater sensors challenges and one of the main challenges is life-time and power consumptions; there are also many studies about improving the battery but there are still lack in the studies of low power-consumption. Thus, more studies are needed to cover this gap.

II. METHODOLOGY

In this thesis, the aim was predicting the trajectory of a sinking underwater sensor by using kinetic equation of sinking object. The sinking object position P will be giving by three time-dependent coordinates (x,y,z).

With respect to position, the velocity and the acceleration equation is written as $\vec{v} = x \vec{i} + y \vec{j} + z \vec{k}$ and $\vec{a} = x \ddot{i} + y \ddot{j} + z \ddot{k}$, respectively. The study took on consideration four forces acting on the sensor as showing in Fig. (1) and described in equations (1) to (4).

The first force is the weight \vec{F}_W , which reads:

$$\vec{F}_W = \rho_s V_s \vec{g}, \quad (1)$$

where ρ_s is the sensor's density, V_s is the sensor's volume and \vec{g} is the terrestrial gravitational acceleration. The sinking sensor is also subjected to the buoyant force \vec{F}_B , which is equal to the weight of the displaced water. It is written as:

$$\vec{F}_B = -\rho_w V_s \vec{g}, \quad (2)$$

where ρ_w is the water density, which can depend on the water depth z , i.e., $\rho_w(z)$. The third force is the one applied by the water current. It writes:

$$\vec{F}_C = C\sigma A_C(\vec{v}_C - \vec{v}_{//}), \quad (3)$$

where C is a constant, σ is a shape factor, A_C is the sensor's cross-section area facing the current, \vec{v}_C is the velocity of the water current and $\vec{v}_{//}$ is the projection of the sensor's velocity in the same plane as the current velocity. More precisely, any upwelling or downwelling effects are neglected [1]. Thus, no current is considered in the vertical direction. The water current is simplified to $\vec{v}_C = v_C^x \vec{i} + v_C^y \vec{j}$. Consequently, $\vec{v}_{//} = \dot{x} \vec{i} + \dot{y} \vec{j}$. The last force is the water-resistant force which is applied normally to the current velocity plane. It reads:

$$\vec{F}_R = -K\rho_w \mu A_R(\vec{v} - \vec{v}_{//}), \quad (4)$$

where K is a constant, μ is a shape factor and A_R is the sensor's cross-section area perpendicular to the current. Applying the second Newton's law on the free-body diagram shown in Fig. (1) yields:

$$\vec{F}_B = (\rho_s - \rho_w)V_s \vec{g} + \alpha_C(\vec{v}_C - \vec{v}_{//}) - \alpha_R(\vec{v} - \vec{v}_{//}) = \rho_s V_s \vec{a}, \quad (5)$$

where $\alpha_C = C\sigma A_C$ and $\alpha_R = K\rho_w \mu A_R$. Projecting equation (5) on the three space directions leads to the following dynamic system equations:

$$\ddot{x} + \frac{\alpha_C}{\rho_s V_s} \dot{x} = \frac{\alpha_C}{\rho_s V_s} v_C^x, \quad (6)$$

$$\ddot{y} + \frac{\alpha_C}{\rho_s V_s} \dot{y} = \frac{\alpha_C}{\rho_s V_s} v_C^y, \quad (7)$$

and

$$\ddot{z} + \frac{\alpha_R}{\rho_s V_s} \dot{z} = \frac{\rho_s - \rho_w}{\rho_s} g. \quad (8)$$

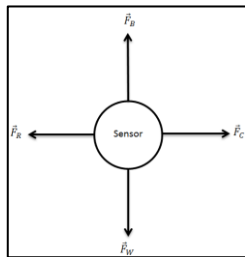


Fig. 1: Free-body diagram.

If the current velocities v_C^x and v_C^y , and the water density ρ_w are constant, the above dynamic system equations turns to a linear system that can be solved analytically [1].

$$x(t) = x(t^0) + v_C^x (t - t^0) + \frac{x(t^0) - v_C^x}{\alpha_C / \rho_s V_s} (1 - e^{-\alpha_C / \rho_s V_s (t - t^0)}), \quad (9)$$

$$y(t) = y(t^0) + v_C^y (t - t^0) + \frac{y(t^0) - v_C^y}{\alpha_C / \rho_s V_s} (1 - e^{-\alpha_C / \rho_s V_s (t - t^0)}), \quad (10)$$

and

$$z(t) = z(t^0) + v_C^z (t - t^0) + \frac{z(t^0) - v_C^z}{\alpha_R / \rho_s V_s} (1 - e^{-\alpha_R / \rho_s V_s (t - t^0)}). \quad (11)$$

Where t^0 is a reference time, $x(t^0)$, $y(t^0)$, and $z(t^0)$ are the

sensor coordinates at t^0 , and $\dot{x}(t^0)$, $\dot{y}(t^0)$, and $\dot{z}(t^0)$ are the sensor velocity components at t^0 .

However,

$$v_C^x = k_1 \lambda (v \sin(k_2 x) \cos(k_3 y) + \cos(2k_1 t)) + k_4, \quad (12)$$

and

$$v_C^y = -\lambda v \cos(k_2 x) \sin(k_3 y) + k_5. \quad (13)$$

where $k_1, k_2, k_3, k_4, k_5, \lambda$ and v are parameters that depend on the underwater environment such as tide and bathymetry [6].

III. RESULTS

We can calculate the position and velocity of a single sensor at any time by knowing the initial position and velocity.

We used a numerical approach to solve the non-linear equations (6) to (8) and we developed a MATLAB code that simulates the location along x and y direction of a single underwater sensor with respect to time in equations (9) and (10) knowing the current velocity defined in equations (12) and (13).

A. Inputs

We have fixed some inputs related to the sensor geometry and other constants as below as well as we have tried changing some other input to simulate different initial conditions and different sea water or sensor characteristics as will be explain in the coming sections.

The sensor's density $\rho_s = 1025 \text{ kg/m}^3$, The sensor's volume $V_s = 0.01 \text{ m}^3$, The terrestrial gravitational $\vec{g} = 9.81 \text{ m/s}^2$, Sea Water Height (Water Depth) $h = 500 \text{ m}$, The water density at top level at $h=0 \text{ m}$ $\rho_t = 1020 \text{ kg/m}^3$, The water density at bottom at $h=500 \text{ m}$ $\rho_b = 1050 \text{ kg/m}^3$, Shape Factors $\sigma = 1$ and $\mu = 1$, Constants $C = 721.7 \text{ N s/m}^3$ and $K = 0.2 \text{ N m}^2 \text{ s/Kg}$, Underwater environment parameters $k_1 = 0.3 \pi$, $k_2 = \pi$, $k_3 = 2 \pi$, $k_4 = 1$, $k_5 = 0.1$, $\lambda = 1$ and $v = 0.3$, Time $t = 3600 \text{ s}$, Time step $\Delta t = t - t^0 = 1 \text{ s}$, Initial time $t^0 = 0 \text{ s}$, Initial position $x(t^0) = 0 \text{ m}$, $y(t^0) = 0 \text{ m}$ and $z(t^0) = 0 \text{ m}$, Initial velocity $\dot{x}(t^0) = 0 \text{ m/s}$, $\dot{y}(t^0) = 0 \text{ m/s}$ and $\dot{z}(t^0) = 0 \text{ m/s}$.

B. Simulate with different time step Δt

As we simulate the equations over a time of one hour $t = 3600 \text{ s}$ with a small, we started by big time-step of $\Delta t = 500 \text{ s}$ and after that simulates with smaller time step. The more the time step got shorter the more the result got more accurate.

We used $\Delta t = 0.1 \text{ s}$ as shortest time step and it will be the reference of the results. We have simulated the programming code with different time step $\Delta t = \{1, 10, 100, 250, 500\} \text{ s}$ and then checked the effect on final position of the sensor at $t = 3600 \text{ s}$ along x and y direction as shown in Fig. 2 and 3 and then calculated the results error with reference to the result at $\Delta t = 0.1 \text{ s}$ as shown in Table I.

As we noticed in Table I, the result error percentage for the time step $\Delta t = 1s$ is negligible comparing to the result for the time step $\Delta t = 0.1s$ and therefore we decided to go forward with the results using the time step of $\Delta t = 1s$.

TABLE I: SENSOR POSITION AT T=3600S FOR DIFFERENT Δt AND THE ERROR VS. $\Delta t = 0.1s$

Δt (s)	x (m)	x error	y (m)	y error
0.1	6938.3	-	357.17	-
1	6939.1	0.01 %	356.48	-0.19 %
10	6942	0.05 %	381.38	6.78 %
100	7010.7	1.04 %	416.78	16.69 %
250	6637.6	-4.33 %	373.26	4.50 %
500	6966.3	0.40 %	123.11	-65.53 %

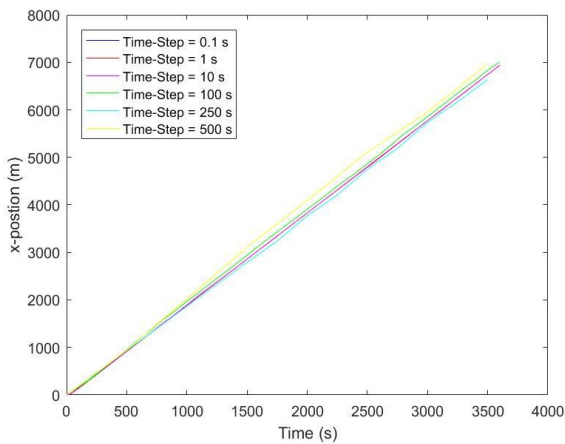


Fig. 2. Effect of Time Step on x-position.

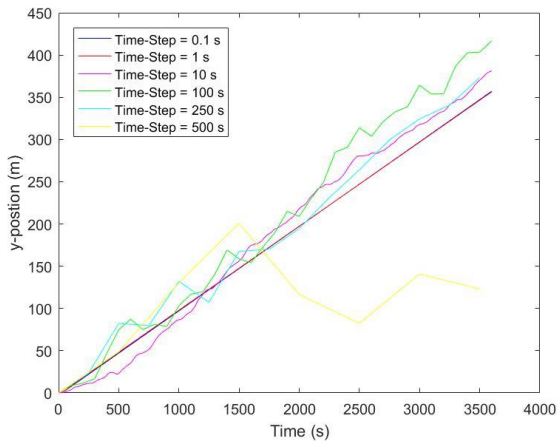


Fig. 3. Effect of Time Step on y-position.

C. Change initial condition of sensor location along x direction

It was very important to check the effect of small error in the initial conditions and if the result will deviate or not. We have started by initial condition of x and y equal to zero and then we have tested different starting point of $x = \{0, 0.01, 0.005, 0.01, 0.05, 1, 5, 10\} m$, the sensor final positions at $t = 3600s$ calculated with reference $x(t^0) = 0$ and $y(t^0) = 0$ as shown in Table II, Fig. 4 and Fig. 5. As explained in previous section, all the result simulated with time step of $\Delta t = 1s$.

As noticed in Table II the result error effected of changing the initial condition along x direction are small with minor

effect.

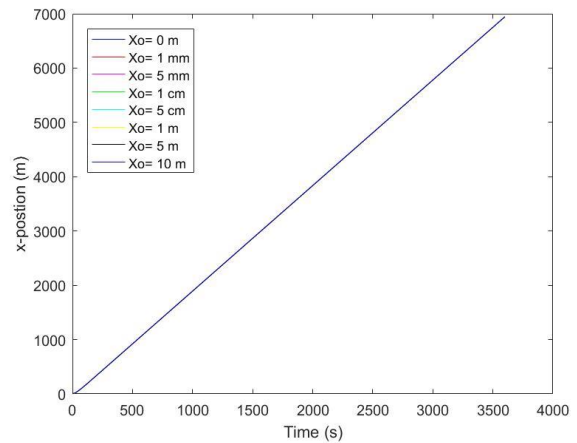


Fig. 4. Effect of different x initial condition on x-position.

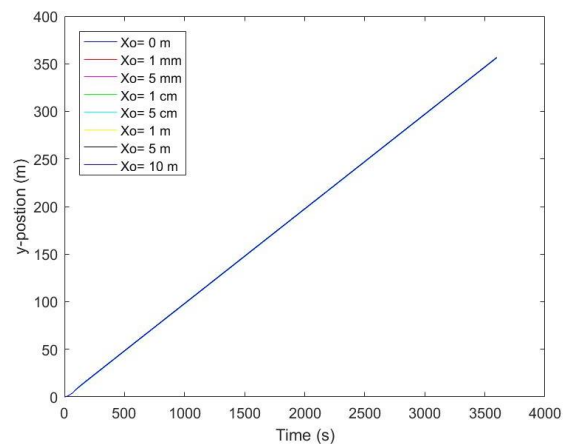


Fig. 5. Effect of different x initial condition on y-position.

TABLE II: DIFFERENT X INITIAL CONDITION EFFECT

$t^0 = 0s$		$t = 3600s$			
$x(t^0)$ (m)	$y(t^0)$ (m)	$x(t)$ (m)	x error	$y(t)$ (m)	y error
0	0	6939.110	-	356.479	-
0.001	0	6939.117	0 %	356.471	0 %
0.005	0	6939.145	0 %	356.441	-0.010 %
0.01	0	6939.179	0.001 %	356.404	-0.020 %
0.05	0	6939.403	0.004 %	356.152	-0.090 %
1	0	6938.439	-0.010 %	356.892	0.120 %
5	0	6942.439	0.048 %	356.892	0.120 %
10	0	6949.11	0.144 %	356.479	0 %

IV. DISCUSSION

As shown in Table II, the error of changing initial condition values of x had relatively small error. In this section we re-simulated the results for the critical path with initial conditions of $x(t^0) = 10m$ and $y(t^0) = 10m$ for the different time step $\Delta t = \{0.1, 1, 10, 100, 250, 500\}s$ and calculated the result error with reference to $\Delta t = 0.1s$ $x(t^0) = 0$ and $y(t^0) = 0$ as shown in Table III.

TABLE III: ERROR CALCULATION OF CRITICAL PATH ALONG X AND Y DIRECTION

Δt (s)	$t^0 = 0 s$		$t = 3600 s$			
	$x(t^0)$ (m)	$y(t^0)$ (m)	$x(t)$ (m)	x error	$y(t)$ (m)	y error
0.1	0	0	6938.3	-	357.17	-
1	0	0	6939.1	0.01 %	356.48	-0.19 %
10	0	0	6942	0.05 %	381.38	6.78 %
100	0	0	7010.7	1.04 %	416.78	16.69 %
250	0	0	6637.6	-4.33 %	373.26	4.50 %
500	0	0	6966.3	0.40 %	123.11	-65.53 %
0.1	10	0	6948.3	0.14 %	357.17	0 %
1	10	0	6949.1	0.16 %	356.48	-0.19 %
10	10	0	6906.2	-0.46 %	354.99	-0.61 %
100	10	0	6993.7	0.80 %	338.04	-5.36 %
250	10	0	6807.3	-1.89 %	248.75	-30.36 %
500	10	0	6939.5	0.02 %	85.64	-76.02 %
0.1	0	10	6938.3	0 %	367.17	2.80 %
1	0	10	6939.1	0.01 %	366.48	2.61 %
10	0	10	6906.7	-0.46 %	357.21	0.01 %
100	0	10	6924.1	-0.20 %	410.94	15.05 %
250	0	10	6663	-3.97 %	441.99	23.75 %
500	0	10	6898.2	-0.58 %	102.95	-71.18 %

V. CONCLUSION

The conclusions drawn from the current investigations may be summarized as follows:

- An analytical model for the mobility of an underwater sensor was developed and resulted to a set of non-linear time-dependent partial differential equations.
- Computer programming simulation developed to predict the motion of underwater sensor.
- The simulation result along x direction found accurate even with partially larger time step while along y direction the model resulted with relatively larger error and deviation in case of the large time step.
- The different between the sensor velocity and the current velocity found neglectable along x direction oppositely along y direction where found bigger difference between the sensor velocity and the current velocity and the error is important to consider.

For future work and based on the result got from this work, we believe the below research areas is important and recommend to consider for future researches.

- Simulation with different time step for x and y direction.
- Simulate the problem for multi sensors.
- Simulate with real initial condition data for the sea density and current velocity.

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