A Fuzzy Inventory Model Considering Imperfect Quality Items with Receiving Reparative Batch and Order

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***Abstract* — This paper presents an inventory model for imperfect quality items with receiving a reparative batch and order overlapping in a fuzzy environment by employing fuzzy triangular numbers. It is assumed that the imperfect items identified by Screening are divided into either scrap or reworkable items. The reworkable items are kept in store until the next items are received. Afterward, the items are returned to the supplier to be reworked. Also, a discount on the purchasing cost is employed as an offer of cooperation from a supplier to a buyer to compensate for all additional holding costs incurred to the buyer. The rework process is error-free. An overlapping order scheme is employed so that the vendor is allowed to use the previous shipment to meet the demand by the inspection period. In the fuzzy model, the graded mean integration method is taken to defuzzify the model and determine its approximation of a profit function and optimal policy. In doing so, numerical examples are rendered to represent the model behavior, and, eventually, the sensitivity analysis is presented.**

***Index Terms* — Inventory, Imperfect quality, Order overlapping, Graded mean integration, Triangular fuzzy number, Screening.**

I. INTRODUCTION

The importance of inventory management systems is growing every day, and many researchers are trying to solve management problems using mathematical models. The economic order quantity (EOQ) model is the basis of advanced inventory systems. By exploring the literature review on inventory systems, it is realized that many efforts have been conducted to provide inventory models in order to eliminate the limitations of the EOQ model. One of the assumptions in the EOQ models is that all the received items are perfect. However, this assumption is not comprehensive for several reasons, including the faulty production process, failure in the process of transportation, etc. So, the effect of imperfect items on inventory systems has become one of the interesting topics for many researchers to provide more practical models. [1] followed by Rosenblatt and Lee, presented the significant connection between imperfect quality and lot sizing. [2] assumed that the imperfect items received in the lot would result in the inspection cost. In addition, [3] studied an EOQ model with the effect of a joint lot sizing and Screening, in which the imperfect items were random variables. Further, [4] investigated an economic production quantity model for defective items with a known probability distribution. Therefore, they assumed that, by the end of the inspection time, the imperfect items were sold as a single batch. In the same year, they did not deviate from the main idea, but pointed out and rectified an existing error in the model, which was devised by Salameh and Jaber. After that, a simple method was proposed by [5] to find the optimum value of production quantity for the model given by Salameh and Jaber. Subsequently, [6] examined the imperfect inventory model, given that the imperfect items were random variables. [7] extended the inventory model presented by Salameh and Jaber, while considering the effect of error in the screening procedure.

[8] further presented an EOQ inventory model considering imperfect quality items. In the same year, [9] suggested an inventory model, in which lot-sizing, defective items, quality control were combined. [10] formulated a new imperfect production inventory model, in which the imperfect items were randomly produced. Currently, [11] considered the effect of imperfect items and deterioration. It should be noted that, in all the above-mentioned models, no shortage has been assumed during the inspection process, which was based on the model by Salameh and Jaber. Since in several successive inspection processes, defective items may have been found to cause a shortage, the supposed assumption could not be correct. This fault was discussed by [12], who concluded that the simple formula could not be found to prevent the occurrence of shortage during the inspection process. Luckily, [13] developed a pragmatic method to overcome this fault. This method, called "an overlapping order scheme," let the vendor use the previous order to meet the demand during the inspection process. This new approach can effectively prevent the occurrence of shortage during the inspection process. Therefore, this idea was incorporated into our model.

Another unrealistic assumption considered in the above models was that imperfect goods could just be sold at their salvage value and could not be reworked. However, many researchers have discovered this fault and incorporated the idea of reworking a part of imperfect items into their models. [14] studied an EOQ model, in which a part of defective items could be used as good items.[15] and many other researchers have also investigated the effect of the reworked process on the inventory models. It should be noted that the above inventory models consider that the reworkable items are sent back to be reworked and returned as the perfect items through the same period; however, in our model, we assumed that reworkable items were kept in the buyer's warehouse until the next shipment arrived. Then, the supplier replaced the reworkable items with the perfect ones and sent them within the next order before the current

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lot was used up. In the present paper, it was assumed that the following lot was received from the supplier as a "reparative batch." Also, in the previous papers, it was assumed that the perfect item holding costs and scrap item holding costs were the same. However, [16], [17] presented an imperfect EOQ inventory model with different holding costs and learning in the inspection.

In all the above models, researchers have only considered all the parameters and variables as crisp values. Although crisp models offer an overview of the approach of inventory systems under various assumptions, they are not able to provide factual terms. As a result, exerting crisp models, in general, can lead to errors in decision-making. Moreover, in crisp models, inventory managers must be flexible in determining the economic lot size to cause uncertainty cost reduction. Furthermore, the use of fuzzy systems for solving inventory problems, rather than using probability systems, generates more appropriate solutions. Fuzzy sets introduced the attention of many researchers in inventory management topics.

[18] developed a fuzzy scheduling inventory model considering a constraint in warehouse capacity. [19] presented an EOQ model for interpreting a fuzzy set theory. [20] developed an EOQ inventory model considering the backorder as a triangular fuzzy number. [21] provided an inventory model without backorder, considering the fuzzy storing cost defuzzified by centroid and signed distances also proposed an imperfect inventory model considering the fuzzy annual demand and fuzzy imperfect rate. [22] studied a fuzzy EOQ inventory model with two-phase trade credits for deteriorating items in the fuzzy sense. Since then, [23], [24] has made significant contributions to controllable lead-time literature. [25] fuzzified lead time components and studied the effect of flexibility in lead time on the distributors. [26] investigated an EPQ model without any shortages by fuzzifying the decision variable and cycle time and proposed the EOQ model considering fuzzy triangular numbers for the demand and lead time.

As it is obvious from the above-mentioned literature, none of the authors has presented an imperfect EOQ inventory model, either scrap or re-workable, along with receiving reparative batch considering various holding costs for perfect and scrap items under fuzzy conditions in the model parameters. Therefore, we tried to eliminate the gap in the literature. In this paper, scrap items were being sold for salvage value by the end of the inspection period. Upon the completion of the screening process, the buyer notifies the supplier of the number of reworkable items; however, unlike some of the previous articles, here, it is assumed that reworkable items are stored in the buyer's warehouse until the next shipment arrives. Then, the supplier replaces the reworkable items with the perfect ones and sends them within the next order before the current lot is exhausted. Totally, the major distinction between this paper and others lies in fuzziness in the model parameter, the various assumptions on imperfect items, employing an overlapping scheme to prevent shortages during the inspection period, discount rate provision of the purchasing cost to maintain a cooperative relationship, and considering receiving reparative.

The structure of this paper is as follows. The problem statement is given in the second section. Next, the mathematical model is presented. Afterward, numerical examples and sensitivity analysis are given. Finally, the conclusion section is provided.

II. Problem Statement

In this section, the problem is introduced with more details. An imperfect EOQ inventory model is presented. All the items received on a shipment are required to be inspected. The imperfect items that are identified through Screening are divided into either scrap or reworkable items. By the end of the inspection period, the scrap items are sold at a price of salvage value. Then, the buyer declares the number of reworkable items; however, unlike some of the previous articles in which reworkable items are assumed to be sent back to the supplier and returned as the perfect items through the same period, the proposed model is assumed that reworkable items are kept in a buyer's warehouse until the next shipment arrives. Then, the supplier replaces the reworkable items with the perfect ones and sends them within the next order before the current lot is exhausted. By doing so, the supplier's costs (e.g., transportation costs) are reduced and, instead, the buyer's costs (e.g., holding costs) are raised. As a result, a coordinated policy should be employed so that economic benefits can be provided for both the buyer and the supplier. Discount on purchase costs can be used as an offer of cooperation from supplier to buyer (i.e., the discount compensates for all additional holding costs incurred to the buyer). Moreover, to eliminate shortages within the inspection period, an "overlapping scheme" is employed: similar to Maddah et al.'s (2010) idea that lets the buyer supply his/her needs from the previous order during the inspection process. Also, it is assumed that the holding costs for scrap items and perfect items are not the same. Besides, the input parameter D is considered a triangular fuzzy number and applies a graded mean integration method as a defuzzification method to obtain the optimum values. In addition, [27], [28] stated that demand is stochastically distributed in its nature in most industries.

The following are the assumptions considered in this paper:

- Item demand is constant over time.
- The input parameter D is the triangular fuzzy number.
- A graded mean integration method is applied as defuzzification so that the optimum value of the profit function in the fuzzy case could be found.
- Shortages are not allowed.
- The holding cost for reworkable items is different from and higher than the holding cost for scrap items.
- A discount on the purchasing cost is applied to make up for the extra holding cost belonging to the buyer.
- An overlapping order scheme is incorporated into the model.
- The demand and screening processes proceed concurrently, but the Reworking process is error-free.

III. Mathematical Modeling

$\widetilde{D}$: Demand per year, nonnegative triangular fuzzy number with parameter (q, r, s)
x: Inspection rate
A: Ordering cost per cycle
r_s: The percentage rate of scrap items (random variable)
r_w: The percentage rate of reworkable items (random variable)
$f(r_s)$: r_s Probability density function
$f(r_w)$: r_w Probability density function
s: Selling value per unit
w: Salvage value per unit
d: Unit inspection cost
h_w: Reworkable or perfect item holding cost rates per unit per cycle
h_s: Scrap item holding cost rate per unit per cycle
α: Discount rate for procurement cost
c: Purchasing cost per unit
t_1: Screening length per cycle
T: Length of cycle
$H_s(Q)$: Scrap item holding cost per cycle
$H_w(Q)$: Perfect or reworkable item holding costs per cycle
$TP(Q)$: Total profit per cycle
$TPU(Q)$: Net profit per unit time
Q: Order size per cycle (decision variable)

Considering that the demand rate D is a fuzzy number; however, other components of the model are all crisp constant. We represent the demand rate by a triangular fuzzy number as given below:

$$\widetilde{D} = (q, r, s) \tag{1}$$

And the membership function is as follows:

$$\mu_{\widetilde{D}}(x) = \begin{cases} \dfrac{x-q}{r-q} & if q \le x \le r \\ \dfrac{s-x}{x-r} & if r \le x \le s \\ 0 & otherwise \end{cases} \tag{2}$$

Fig. 1 presents the behavior of the proposed model per cycle. The 100% inspection process is finished at the time t_1. To avoid shortages, the overlapping scheme is used, and it is supposed that the demand by the screening time is at least the same as the number of perfect quality items. It means that, for $0 \le t \le t_1$:

$$x t_1 (1 - r_s - r_w) \ge \widetilde{D} t_1 \tag{3}$$

which yields:

$$x \ge \frac{\widetilde{D}}{(1 - r_s - r_w)} \tag{4}$$

The goal is to obtain Q that maximizes the total profit per year, $TP(Q)$, expressed by:

$$TP(Q) = TR(Q) - TC(Q) \tag{5}$$

where $TR(Q)$ denotes the revenue per cycle, and $TC(Q)$ denotes the total cost per cycle. $TR(Q)$ is obtained through the sale of good items and scrap items, i.e.:

$$TR(Q) = sQ(1 - r_s) + \omega Q r_s \tag{6}$$

$TC(Q)$ includes the following four costs:

$$TC(Q) = OC + SC + PC + HC \tag{7}$$

where C denotes the ordering cost per cycle ($OC = A$), SC denotes the screening cost per cycle ($SC = dQ$), PC denotes the purchasing cost per cycle ($PC = cQ(1-\alpha)$), and HC denotes the holding cost per cycle, which includes the scrap item holding cost per cycle, $H_s(Q)$, and reworkable or perfect item holding cost per cycle, $H_w(Q)$. $H_s(Q)$ can be obviously calculated using Fig. 1, as shown in the shaded area:

$$H_s(Q) = h_s \left(\frac{Q^2 r_s}{2x} \right) \tag{8}$$

To compute $H_w(Q)$, the total inventory quantity per cycle should be calculated. According to Fig. 1, it is clear that the sum of the areas of ΔZBC, ΔBGR, GIJR, and ΔRJF minus ΔDEF can express the total inventory quantity per cycle. The area of ΔZBC is the same as that of ΔDEF; therefore, we have:

$$\widetilde{V} = \underbrace{\frac{Q^2 r_s}{2x}}_{\triangle BGR} + \underbrace{\frac{Q^2(1-r_s)}{x}}_{\square GIJR} + \underbrace{\frac{(Q(1-r_s))^2}{2\widetilde{D}}}_{\triangle RJF} \tag{9}$$

Hence, the holding cost $\widetilde{H}_w(Q)$ is as follows:

$$\widetilde{H}_w(Q) = h_w \times V = h_w \left(\frac{Q^2 r_s}{2x} + \frac{Q^2(1-r_s)}{x} + \frac{(Q(1-r_s))^2}{2\widetilde{D}} \right) \tag{10}$$

Thus:

$$T\tilde{C}(Q) = A + dQ + cQ\left(1 - \frac{r_w Q}{\widetilde{D}}\right) + h_s\left(\frac{Q^2 r_s}{2x}\right) + h_w\left(\frac{Q^2 r_s}{2x} + \frac{Q^2(1-r_s)}{x} + \frac{(Q(1-r_s))^2}{2\widetilde{D}}\right) \tag{11}$$

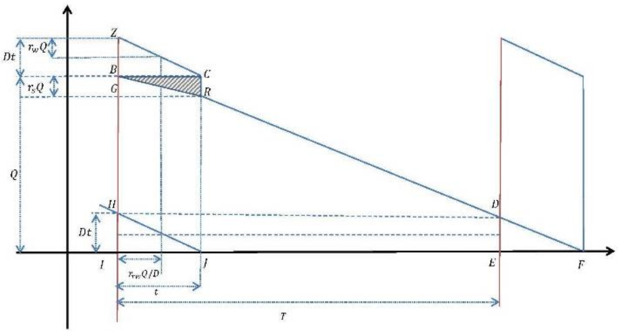

Fig. 1. Inventory model.

Through items simplification, the expression for a total cost per cycle can be calculated by:

$$T\tilde{C}(Q) = A + dQ + cQ\left(1 - \frac{r_w Q}{\widetilde{D}}\right) + (h_s + h_w)\left(\frac{Q^2 r_s}{2x}\right) + h_w\left(\frac{Q^2(1-r_s)}{x}\right) + h_w\left(\frac{(Q(1-r_s))^2}{2\widetilde{D}}\right) \tag{12}$$

By substituting Eqs. (12) and (6) in Eq. (5), the total profit per cycle is obtained by:

$$T\tilde{P}(Q) = sQ(1-r_s) + \omega Q r_s - A - dQ - cQ\left(1 - \frac{r_w Q}{\tilde{D}}\right) - (h_s + h_w)\left(\frac{Q^2 r_s}{2x}\right) - h_w\left(\frac{Q^2(1-r_s)}{x}\right) - h_w\left(\frac{(Q(1-r_s))^2}{2\tilde{D}}\right) \quad (13)$$

Furthermore, it is considered that the expected value of $T\tilde{P}(Q)$ (i.e., $E[T\tilde{P}(Q)]$) is calculated, in which the expected values ($E[1-r_s]$, $E[r_s]$, and $E[r_w]$) are used instead of $1-r_s$, r_s, and r_w, respectively. The expected net profit per unit time is calculated by applying the renewal reward theorem (Ross, 1996) (i.e., dividing $T\tilde{P}(Q)$ by the cycle length, $\tilde{T} = \frac{(1-r_s)Q}{\tilde{D}}$) as follows:

$$E[T\tilde{P}U(Q)] = \frac{\tilde{D}\left(s(1-E(r_s)) + \omega E(r_s) - c\left(\frac{1-E(r_w)Q}{\tilde{D}}\right) - d\right) - \left(\frac{A\tilde{D}}{Q}\right)}{1-E(r_s)} - \frac{Q}{2(1-E(r_s))}\left(\frac{\tilde{D}(2h_w - h_w E(r_s) + h_s E(r_s))}{x} + h_w(E(1-r_s)^2)\right) \quad (14)$$

Now, we consider:

$$u = \frac{1}{1-E(r_s)}\left\{s(1-E(r_s)) + \omega E(r_s) - c - d - \frac{A}{Q}\right\}$$
$$W = \frac{Q}{2(1-E(r_s))}\left\{\frac{(2h_w - h_w E(r_s) + h_s E(r_s))}{x}\right\} \quad (15)$$
$$X = \frac{h_w(E(1-r_s)^2)}{2(1-E(r_s))}$$

Hence, by substituting Eq. (15) in Eq. (14), the annual fuzzy net profit function is illustrated by:

$$E(\tilde{T}PU(Q)) = \tilde{D}(u) + \frac{cE(r_w)}{1-E(r_s)}Q - \tilde{D}(W) - Q(X) \quad (16)$$

Therefore, the annual fuzzy net profit function is illustrated by a nonnegative triangular fuzzy number as follows:

$$E(\tilde{T}PU(Q)) = (a_1, b_1, c_1) \quad (17)$$

where a_1, b_1, c_1 can be obtained below. According to Eq. (16), we have:

$$E\left(\tilde{T}PU(Q)\right) - \frac{cE(r_w)}{1-E(r_s)}Q + Q(X) = \tilde{D}(u - W)$$
$$\Rightarrow \tilde{D} = \frac{E(\tilde{T}PU(Q)) - \left(\frac{cE(r_w)}{1-E(r_s)}\right)Q + Q(X)}{u - W} \quad (18)$$

Using Eqs. (18) and (17), we have:

$$\text{for} \tilde{D} \le a \Rightarrow \frac{E(\tilde{T}PU(Q)) - \left(\frac{cE(r_w)}{1-E(r_s)}\right)Q + Q(X)}{u - W} \le a$$
$$\Rightarrow E(\tilde{T}PU(Q)) \le \underbrace{a(u - W) + \frac{cE(r_w)}{1-E(r_s)}Q - Q(X)}_{a_1} \quad (19)$$

$$\Rightarrow \underbrace{a(u - W) + \frac{cE(r_w)}{1-E(r_s)}Q - Q(X)}_{a_1} \le E(\tilde{T}PU(Q)) \le \underbrace{b(u - W) + \frac{cE(r_w)}{1-E(r_s)}Q - Q(X)}_{b_1} \quad (20)$$

$$\text{for} b \le \tilde{D} \le c \Rightarrow b \le \frac{E(\tilde{T}PU(Q)) - \left(\frac{cE(r_w)}{1-E(r_s)}\right)Q + Q(X)}{u - W} \le c$$
$$\Rightarrow \underbrace{b(u - W) + \frac{cE(r_w)}{1-E(r_s)}Q - Q(X)}_{b_1} \le E(\tilde{T}PU(Q)) \le \underbrace{c(u - W) + \frac{cE(r_w)}{1-E(r_s)}Q - Q(X)}_{c_1} \quad (21)$$

Thus, $E(\tilde{T}PU(Q))$ is a triangular fuzzy number with three points (a_1, b_1, c_1) as follows:

$$E(\tilde{T}PU(Q)) = \begin{cases} a_1 = a(u - W) + \frac{cE(r_w)}{1-E(r_s)}Q - Q(X) \\ b_1 = b(u - W) + \frac{cE(r_w)}{1-E(r_s)}Q - Q(X) \\ c_1 = c(u - W) + \frac{cE(r_w)}{1-E(r_s)}Q - Q(X) \end{cases} \quad (22)$$

$E(\tilde{T}PU(Q))$ is deffuzified by employing the graded mean integration method as follows:

$$P\left(E(\tilde{T}PU(Q))\right) = \frac{1}{6}(a_1 + 4b_1 + c_1) = \frac{1}{6}\begin{pmatrix} \left\{a(u - W) + \frac{cE(r_w)}{1-E(r_s)}Q - Q(X)\right\} + \\ 4\left\{b(u - W) + \frac{cE(r_w)}{1-E(r_s)}Q - Q(X)\right\} + \\ \left\{c(u - W) + \frac{cE(r_w)}{1-E(r_s)}Q - Q(X)\right\} \end{pmatrix} \quad (23)$$

The target is to maximize $P\left(E(\tilde{T}PU(Q))\right)$. Because $P\left(E(\tilde{T}PU(Q))\right)$ is concave at Q, $\frac{\partial^2 E(\tilde{T}PU(Q))}{\partial^2 Q} = \frac{-2A\tilde{D}}{Q^3(1-E(r_s))} < 0$, then the optimum lot size Q^*can be calculated by differentiating $P\left(E(\tilde{T}PU(Q))\right)$ With respect to Q and setting, the partial derivatives are equal to zero.

$$\frac{\partial P(E(\tilde{T}PU(Q)))}{\partial Q} = \frac{1}{6}(a + 4b + c)\left(\frac{\partial u}{\partial Q} - \frac{\partial W}{\partial Q}\right) + \frac{cE(r_w)}{1-E(r_s)} - X = 0 \quad (24)$$

$$\frac{\partial u}{\partial Q} = \frac{\partial}{\partial Q}\left(\frac{\left\{s(1-E(r_s)) + \omega E(r_s) - c - d - \frac{A}{Q}\right\}}{1-E(r_s)}\right) = \frac{A}{Q^2} \times \frac{1}{1-E(r_s)}$$
$$\frac{\partial W}{\partial Q} = \frac{\partial}{\partial Q}\left(\frac{Q\left\{\frac{(2h_w - h_w E(r_s) + h_s E(r_s))}{x}\right\}}{2(1-E(r_s))}\right) = \frac{\frac{(2h_w - h_w E(r_s) + h_s E(r_s))}{x}}{2(1-E(r_s))} \quad (25)$$

Hence, substituting Eq. (25) in Eq. (24):

$$\frac{1}{6}(a + 4b + c) + \left(\frac{A}{Q^2}\frac{1}{1-E(r_s)} - \frac{2h_w - h_w E(r_s) + h_s E(r_s)}{2x(1-E(r_s))}\right) + \frac{cE(r_w)}{1-E(r_s)} - \frac{h_w(E(1-r_s)^2)}{2(1-E(r_s))} = 0 \quad (26)$$

By simplifying Eq. (26), Eq. (27) can be obtained by:

$$\frac{A}{Q^2} = \frac{-6cE(r_w)}{(a+4b+c)} + \frac{3h_w(E(1-r_s)^2)}{(a+4b+c)} + \quad (27)$$

$$\frac{\left\{\frac{(2h_w - h_w E(r_s) + h_s E(r_s))}{x}\right\}(a+4b+c)}{2(a+4b+c)}$$

$$Q^* = \sqrt{\frac{2A(a+4b+c)}{(a+4b+c)\left\{\frac{(2h_w - h_w E(r_s) + h_s E(r_s))}{x}\right\} - 12cE(r_w) + 6h_w(E(1-r_s)^2)}} \quad (28)$$

The optimum annual total profit $P\left(E(\tilde{T}PU(Q))\right)$ is obtained by the direct substitution of Eq. (28) in Eq. (23). Note that, when the input parameter *D* is a real number, that is *a*=*b*=*c*=*D*, when the screening rate is large enough, that is the inspection process is finished simultaneously by the receiving an order, and finally, when items are categorized as only perfect or imperfect (no reworkable items so that no discount on purchasing cost), Q^* in Eq. (28) is equivalent to:

$$Q^* = \sqrt{\frac{2AD}{h_w(E(1-r_s)^2)}} \quad (29)$$

It shows that the proposed model is accurate. In addition, it should be noted that, when the input parameter D is a real number, that is a=b=c=D, and if all items are assumed to be perfect, our model becomes equivalent to the EOQ inventory model.

IV. Numerical Study

In this section, the behavior of our model is investigated by applying numerical examples, and the impact of applying a fuzzy case into the model is also investigated. Assume the following values and the input parameters for an inventory model in the crisp case: *A* = 100 $ per cycle, *D* = 50,000 per unit per year, *x* = 175200 per year per unit, h_w = 5$ per year per unit, *s* = 50$ per unit, *d* = 0.5$ per unit, *w* = 20 per unit, h_s = 2$ per year per unit, *c* = 25$ per unit. Also, $E(r_s) = 0.02 E(r_w) = 0.05 E[(1-r_s)^2] = 0.9605$

The optimum lot size $Q^*$and the optimal annual total profit $E[TPU(Q)]$ of a crisp case, in which *a*=*b*=*c*=*D*=50000, can be derived easily from Eqs. (28) and (23), respectively. It is obtained that $Q^* = 1395 E[TPU(Q)]^* = 1212072$. Some fuzzy triangular numbers are assigned for the input parameter $\tilde{D}$ in Table 1 to illustrate the fuzzy model developed in Section 3. Then, by using the GMI method, the defuzzified values are specified. The defuzzified values and the corresponding percentage difference from the crisp values (denoted by p_D for the component $\tilde{D}$) are also shown in Table 1. For each set of triangular fuzzy numbers, the optimal lot size $Q^*$and the optimal annual total profit $P\left(E(\tilde{T}PU(Q))\right)$ are derived from Eqs. (28) and (23). The findings are summarized in Table 2. This table represents variations in $Q^*$and annual net profit $P\left(E(\tilde{T}PU(Q))\right)$due to fuzziness in parameter *D*., It shows that the optimal values for the expected net profit are fully sensitive to increasing percentage changes in parameter *D*'s fuzziness level, while optimal order quantities are comparatively insensitive to increase percentage changes in parameter *D*'s fuzziness. Note that the percentage changes in the expected net profit are almost the same as the percentage changes of parameter *D* at different levels, whereas the lot size order quantity changes marginally. Besides, a one-way sensitivity analysis is conducted to determine the impact of other problem parameters on $Q^*$and $P\left(E(\tilde{T}PU(Q))\right)$. In the numerical examples, the value of one parameter is changed at a time, while the values of the others are not changed. Table 3 shows the values used in the sensitivity analysis for different problem parameters. Then, the optimal order quantity and the annual total net profit are calculated using the values shown in Table 3. The corresponding values are presented in Table 4.

TABLE I: Triangular fuzzy numbers for the parameter $\tilde{D}$

$\tilde{D}$	$p(\tilde{D})$	p_D
(5,000; 34,250; 68,000)	35,000	-30
(12,000; 37,500; 78,000)	40,000	-20
(20,000; 45,000; 70,000)	45,000	-10
(29,000;52,000; 93,000)	55,000	10
(42,000;61,000; 94,000)	60,000	20
(33,000; 61,500;111,000)	65,000	30

TABLE II: %Change in optimum values from the crisp

Q^*	% change in Q^*	$P\left(E(\tilde{T}PU(Q))\right)$	% change in $P\left(E(\tilde{T}PU(Q))\right)$
1277.64	-8	848731.233	-30
1322.81	-5	970116.010	-20
1361.45	-2	1091503.127	-10
1424.23	2	1334281.969	10
1465.76	5	1536600.692	27
1473.15	6	1577064.666	30

TABLE III: Experimental values for the example

Parameter	Base value	Experimental values		
x	175200	87600	175200	262800
h_w	5	2.5	5	10
h_s	2	1	2	4
A	100	50	100	200
d	0.5	0.25	0.5	1
s	50	25	50	100
c	25	12.5	25	50
w	20	10	20	40
$E(r_s)$	0.02	0.01	0.02	0.03
$E(r_w)$	0.05	0.025	0.05	0.075

TABLE IV: Order quantity and expected net profit per unit

x	Q	$P\left(E(\tilde{T}PU(Q))\right)$	h_w	Q	$P\left(E(\tilde{T}PU(Q))\right)$
87600	1119.7	1210274.6	2.5	2746.9	1215672.9
175200	1394.9	1212072	5	1394.9	1212072
262800	1544.2	1212779.7	10	885	1207858
A	Q	$P\left(E(\tilde{T}PU(Q))\right)$	d	Q	$P\left(E(\tilde{T}PU(Q))\right)$
50	986.4	1214215.1	0.25	1394.9	1224827.6
100	1394.9	1212072	0.5	1394.9	1212072
200	1972	1209042.5	1	1394.9	1186562.3
h_s	Q	$P\left(E(\tilde{T}PU(Q))\right)$	s	Q	$P\left(E(\tilde{T}PU(Q))\right)$
1	1395.7	212076.6	25	1394.9	-37927.4
2	1394.9	1212072	50	1394.9	1212072
4	1393.4	1212064.4	100	1394.9	3712072.5
c	Q	$P\left(E(\tilde{T}PU(Q))\right)$	w	Q	$P\left(E(\tilde{T}PU(Q))\right)$
12.5	1251.1	1848986.5	10	1394.9	1201868.5
25	1394.9	1212072	20	1394.9	1212072
50	1946.5	-61364.6	40	1394.9	1232480.7
$E(r_w)$	Q	$P\left(E(\tilde{T}PU(Q))\right)$	$E(r_s)$	Q	$P\left(E(\tilde{T}PU(Q))\right)$
0.025	1251.1	121123.3	0.01	1393.8	1214974.9
0.05	1394.9	1212072	0.02	1394.9	1212072
0.075	1603.5	1213024	0.03	1396.1	1209110.4

Figs. 2 and 3 display a tornado diagram as a graphical result of the sensitivity analysis. These represent how the order quantity and the annual total net profit are changing, while the model parameters are independently varying from their low value to the high ones. The length of each bar in the diagram shows the extent to which the optimal order quantity and the annual net profit are sensitive to the bar's corresponding model parameter. It can be observed from Fig. 2 that the model's parameters with the greatest impact on the optimum order size are h_w. As other parameters have their base values, the perfect or reworkable item holding cost rate per cycle is differing from 2.5 to 10, while the value of the order quantity changes from 2746.9 to 885. This finding shows that a larger amount of $h_w$can highly affect the order quantity. Moreover, it can be observed from Fig. 3 that the model's parameters with the greatest impact on the annual net profit are the unit screening cost. As other parameters have their base values, when d varied from 0.25 to 1, the value of the annual net profit changes from 1224827 to 1186562. As a result, the values of these parameters should be carefully estimated because they have the most significant impact on the model's cost.

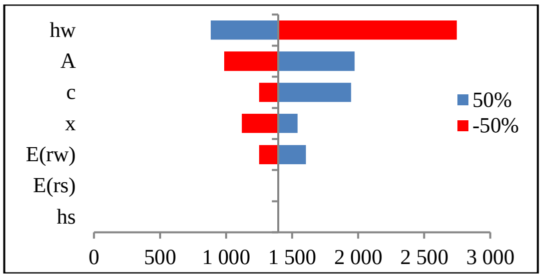

Fig. 2. Tornado diagram for the order quantity.

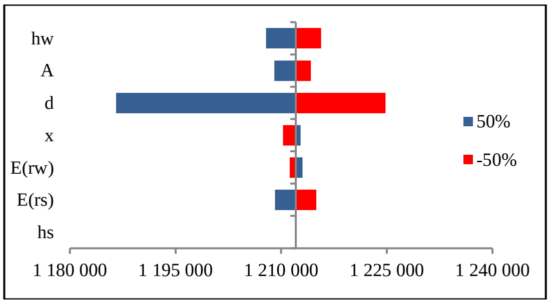

Fig. 3. Tornado diagram for the total annual profit.

V. CONCLUSION

As it is known, the input parameters of the EOQ inventory problem cannot be described precisely in a real situation, or it may be uncertain because of some uncontrolled factors. Therefore, approximate solution approaches have been represented for the explanation of a series of practical inventory problems. Fuzzy methodologies provide a helpful approach to model ambiguity in human recognition and decision-making. Uncertainties defined by imprecise factors can be illustrated by fuzzy sets. Thus, in the current paper, our goal is to propose the fuzzy inventory model with defective items considering reparative batch and order are overlapping. In this model, the input parameter (D) is considered the fuzzy number to defuzzify the proposed model and determine the approximation of annual profit in the fuzzy sense, and we apply the graded mean integration method. Then, the optimal order quantity is calculated to maximize the total profit. The model is solved for triangular fuzzy numbers. It is shown that the EOQ model, as well as the general models, are just some special cases of our model. In so doing, numerical examples are rendered to represent the model behavior, and then the results of the crisp and fuzzy models are compared with each other. It should be noted that the optimal values of the annual net profit are quite sensitive to increasing percentage changes in the parameter D's fuzziness level, while an optimal order quantity is comparatively insensitive to increase percentage changes in the parameter D's fuzziness level. The percentage change in the annual net profit is almost the same as the percentage change in the fuzziness level, while the order size changes slightly. A one-way sensitivity analysis is presented to assess the effect of other problem parameters on the order quantity and annual total net profit and to display the sensitivity analysis results graphically as a tornado diagram. To increase the scope of our analysis, the model presented in this paper can be extended in several ways. For example, it can be incorporated with deteriorating items.

REFERENCES

[1] EL Porteus (1986) Optimal lot sizing, process quality improvement, and setup cost reduction. Operations Research, 34,137–144.
[2] RL Schwaller (1988) EOQ under inspection costs. Production and Inventory Management Journal, 29, 22–24.
[3] XIN Zhang, Y Gerchak (1990) Joint lot sizing and inspection policy in an EOQ model with random yield. IIE Transactions, 22(1), 41-47.
[4] PA Hayek, MK Salameh (2001) Production lot sizing with the reworking of imperfect quality items produced. Production Planning & Control, 12, 584–590.
[5] LE Cárdenas-Barrón, B Sarkar, G Treviño-Garza (2013b) An improved solution to the replenishment policy for the EMQ model with rework and multiple shipments. Applied Mathematical Modelling 37:5549–5554.
[6] S Papachristos, I Konstantaras (2006) Economic ordering quantity models for items with imperfect quality. International Journal of Production Economics, 100, 148–154.
[7] M Khan, MY Jaber, AL Guiffrida, S Zolfaghari. (2011). A review of the extensions of a modified EOQ model for imperfect quality items. International Journal of Production Economics, 132(1), 1–12.
[8] MY Jaber, S Zanoni, LE Zavanella (2013) An entropic economic order quantity (EnEOQ) for items with imperfect quality. Applied Mathematical Modelling, 37(6), 3982–3992.
[9] L Moussawi-Haidar, M Salameh, W Nasr (2013) An instantaneous replenishment model under the effect of a sampling policy for defective items. Applied Mathematical Modelling, 37(3), 719–727.
[10] M Karimi-Nasab, K Sabri-Laghaie (2014) Developing approximate algorithms for EPQ problem with process compressibility and random error in production/inspection. International Journal of Production Research, 52(8), 2388-2421.
[11] L Moussawi-Haidar, M Salameh, W Nasr (2014) Effect of deterioration on the instantaneous replenishment model with imperfect quality items. Applied Mathematical Modelling, 38(24), 5956–5966.
[12] S Papachristos, I Konstantaras (2006) Economic ordering quantity models for items with imperfect quality. International Journal of Production Economics, 100, 148–154.
[13] B Maddah, MK Salameh, L Moussawi-Haidar (2010) Order overlapping: a practical approach for preventing shortages during Screening. Computer Industrial Engineering, 58, 691–695.
[14] HF Yu, WK Hsu, WJ Chang (2012) EOQ model where a portion of the defectives can be used as perfect quality. International Journal of Systems Science, 43, 1689–1698.
[15] LY Ouyang, CJ Chuang, CH Ho, CW Wu (2014) An integrated inventory model with quality improvement and two-part credit policy. TOP, 22(3), 1042-106.
[16] MIM Wahab, MY Jaber (2010) Economic order quantity model for items with imperfect quality, different holding costs, and learning effects: A note. Computers & Industrial Engineering, 58, 186–190.

[17] H Tahami, A Mirzazadeh, A Arshadi-Khamseh, & A Gholami-Qadikolaei. (2016). A periodic review integrated inventory model for buyer's unidentified protection interval demand distribution. Cogent Engineering, 3(1).
[18] G Sommer (1981). Fuzzy inventory scheduling. Applied systems and cybernetics (pp. 3052–3060). New York: Pergamon Press.
[19] KS Park (1987). Fuzzy-set theoretic interpretation of economic order quantity. IEEE Transactions on Systems, Man, and Cybernetics, SMC-17, 1082–1084.
[20] SC Chang, JS Yao, HM Lee (1998). Economic reorder point for fuzzy backorder quantity. European Journal of Operational Research, 109, 183–202.
[21] Chang HC (2004). An application of fuzzy sets theory to the EOQ model with imperfect quality items. Computers and Operations Research, 31, 2079–2092.
[22] GC Mahata, P Mahata. (2011). Analysis of a fuzzy economic order quantity model for deteriorating items under retailer partial trade credit financing in a supply chain. Mathematical and Computer Modelling, 53, 1621–1636.
[23] H Tahami, A Mirzazadeh, & A Gholami-Qadikolaei. (2019). Simultaneous control on lead time elements and ordering cost for an inflationary inventory-production model with mixture of normal distributions LTD under finite capacity. RAIRO-Oper. Res., 53(4), 1357–1384.
[24] H Fakhravar. (2020). Quantifying Uncertainty in Risk Assessment using Fuzzy Theory. ArXiv E-Prints, arXiv:2009.09334.
[25] KM Björk, C Carlsson (2005). The outcome of imprecise lead times on the distributors. In Proceedings of the 38th Annual Hawaii International Conference on System Sciences (HICSS05), p. 81a (10 pp.).
[26] KM Björk (2009). An analytical solution to a fuzzy economic order quantity problem. International Journal of Approximate Reasoning, 50, 485–493.
[27] S Yahoodik, H Tahami, J Unverricht, Y Yamani, H Handley, & D Thompson. (2020). Blink Rate as a Measure of Driver Workload.
[28] H Tahami, H Fakhravar. (2020) Multilevel Reorder Strategy-based Supply Chain Model, 5th North American Conference on Industrial Engineering and Operations Management (IEOM), Michigan, USA.

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