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# Lateral-Torsional Buckling Strength of Two-Span Glass Fiber Reinforced Polymer Beams

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***Abstract*** — **This paper presents the outcome of a study of two-span glass fiber reinforced polymer (GFRP) I-section beams susceptible to lateral-torsional buckling when subjected to gradually increasing concentrated vertical load(s) in the presence of two different types of lateral bracing schemes. It is found that loading one span results in a smaller buckling load as compared with the cases with loading in both spans regardless of the type of bracing scheme used. Also, the study shows that the addition of midspan braces for the GFRP beams results in up to 5.5 times increase in the buckling load capacity.**

***Index Terms*** — **Glass Fiber Reinforced Polymer, I-Beams, Lateral-Torsional Buckling, Bracing.**

## I. INTRODUCTION

Pultruded glass fiber reinforced polymer (GFRP) I-section beams are often susceptible to lateral-torsional buckling when subjected to gradually increasing flexural loads about the cross-sectional major axis. The authors have previously published studies on lateral-torsional buckling behavior of laterally unbraced single-span GFRP beams [1], [4], [6]. In [3], the authors presented a practical solution for avoiding lateral-torsional buckling of single-span I-section beams by addition of lateral bracing. Amponin and Razzaq [5] presented a theoretical and experimental study of a three-span FRP beam under a gradually increasing concentrated load at the center of the middle span. Qiao et al [7] presented a lateral-torsional buckling study of GFRP I-beams including sectional distortions. Nguyen et al. [8] presented a lateral-torsional buckling design procedure for FRP beams. Corria et al studied both buckling and post-buckling behavior of I-section GFRP beams.

The present paper summarizes the outcome of an investigation of a two-span GFRP I-section beam with two different types of bracing systems. The effectiveness of the bracing systems is studied for two different types of loading conditions.

## II. BRACING AND LOADING CONDITIONS

Figure 1 shows four different cases of two-span beam problems investigated in this paper with various lateral bracing schemes and gradually increasing load W applied in one or both spans. In this figure, Case 1 represents a beam with a single concentrated load W at point D and lateral braces provided at the supports only, that is, at A, B, and C. The beam shown as Case 2 has the same brace locations as for Case 1, however, it is subjected to a pair of loads each of magnitude W. Cases 3 and 4 have braces not only at the supports but also at the middle of each span. Furthermore, the beam in Case 3 is loaded only in span AB while that in Case 4 has loads in both spans. Figure 2 shows the a typical cross section before and after lateral-torsional buckling, where u, v, and β are lateral, vertical, and torsional displacements.

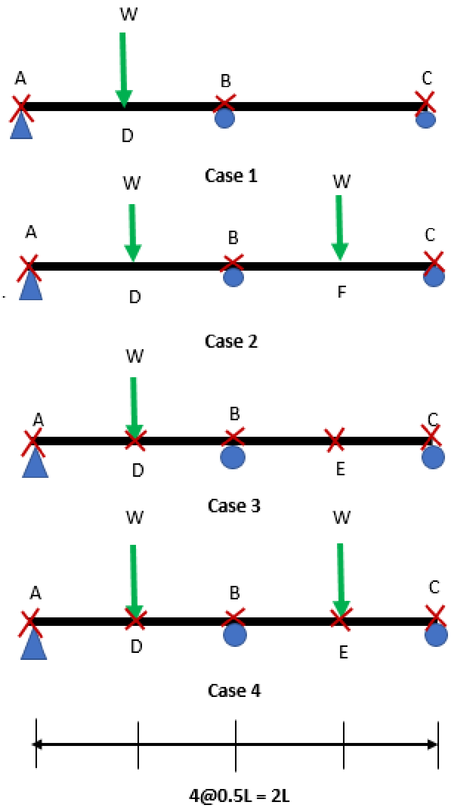

Fig. 1. Two-span beam with bracing and loading conditions.

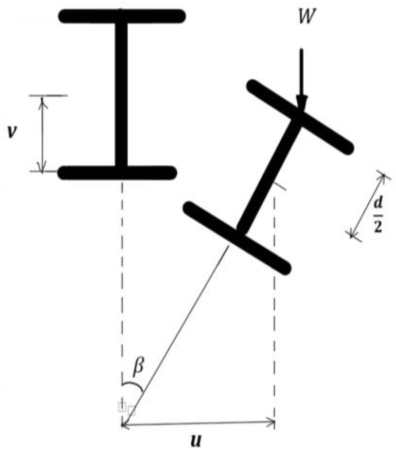

Fig. 2. Cross Section and Displacements.

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## III. LATERAL-TORSIONAL BUCKLING STRENGTH ANALYSIS

The lateral-torsional buckling strength, $W_{LTB}$, for each of the Cases 1 through 4 is based on the following buckling moment expression [2]:

$$M_n = C_b \sqrt{\frac{\pi^2 I_y D_J E_{L,f}}{L_b^2} + \frac{\pi^4 E_{L,f}^2 I_y C_w}{L_b^4}} \qquad (1)$$

in which:

$L_b$ = Unbraced length between consecutive braces, in.
$b_f$ = Full width of the flange, in.
$h$ = Full height of the member, in.
$t_f$ = Thickness of the flange, in.
$t_w$ = Thickness of the web, in.
$k_r$ = Rotational spring constant, kip/rad
$E_{L,f}$ = Characteristic longitudinal modulus of the flange, ksi
$C_b$ = Moment modification factor for unsupported spans with both ends braced
$D_J$ = Torsional rigidity of an open section $=G_{LT}\sum\frac{1}{3}b_i t_i^3$, $kip-in.^2$
$C_\omega$ = Warping constant = $\frac{t_f h^2 b_f^3}{24}$, $in.^6$
$I_y$ = Moment of Inertia about the minor axis, $in^4$

In Equation 1, the $C_b$ values are based on the following expression (Reference 2):

$$C_b = 12.5M_{max}/(2.5M_{max} + 3M_A + 4M_B + 3M_C) \qquad (2)$$

In this exression, $M_{max}$ is the absolute value of the maximum bending moment, and $M_A$, $M_B$, and $M_C$ are the absolute values of the quarter-point moments between any consecutive pair of lateral braces.

Table 1 presents the lateral-torsional buckling analysis results for Cases 1 through 4 based on L = 120.0 in. for an I-section with h = 4.0 in., $b_f$ = 2.0 in., $t_f = t_w$ = 0.25 in., $E_{L,f}$ = 2,550 ksi, and $G_{LT}$ = 420 ksi. In this table, $C_b$ values based on Equation 2 for Cases 1 through 4 are given for each segment. To achieve this, the moment values on the right hand side of Equation 2 are based on in-plane bending moment diagrams. The second-last column in the table lists $M_n$ values computed using Equation 1. The last column in the table lists $W_{LTB}$ based on the $M_n$ values equated with the absolute magnitude of the largest bending moment for each related case.

TABLE 1. LATERAL-TORSIONAL BUCKLING ANALYSIS RESULTS

| Case | Segment | $C_b$ | $L_b$ (in.) | $M_n$ (kip-in.) | $W_{LTB}$ (kips) |
|---|---|---|---|---|---|
| **1** | AB | 1.42 | 120 | 4.696 | 0.193 |
| **1** | BC | 1.67 | 120 | 5.523 | 0.227 |
| **2** | AB | 1.42 | 120 | 4.696 | 0.209 |
| **2** | BC | 1.42 | 120 | 4.696 | 0.209 |
| **3** | AD | 1.67 | 60 | 12.947 | 1.062 |
| **3** | DB | 2.17 | 60 | 16.823 | 1.380 |
| **3** | BE | 1.25 | 60 | 9.691 | 0.796 |
| **3** | EC | 1.67 | 60 | 12.947 | 1.062 |
| **4** | AD | 1.67 | 60 | 12.947 | 1.150 |
| **4** | DB | 2.02 | 60 | 15.660 | 1.392 |
| **4** | BE | 2.02 | 60 | 15.660 | 1.392 |
| **4** | EC | 1.67 | 60 | 12.947 | 1.150 |

## IV. GOVERNING BUCKLING LOADS

For Case 1, segments AB and BC result in buckling loads of 0.193 kips and 0.227 kips, respectively, thus the governing value of $W_{LTB}$ is 0.193 kips. The buckling load for Case 2 is 0.209 kips. For Case 3, segment BE provides the smallest buckling load of 0.796 kips. Lastly, the buckling load of 1.150 kips for Case 4 is simultaneously governed by segments AD and EC.

## V. CONCLUSIONS

Based on the lateral-torsional buckling analysis of the two-span GFRP beams shown in Figure 1, the following conclusions are drawn:

1. Loading one span results in a smaller buckling load as compared with the cases with loading in both spans regardless of the type of bracing scheme used. This is evident from the fact that the beam in Case 1 buckled at a load equal to 92.3 percent of the load for Case 2, and that in Case 3 buckled at a load of 69.2 percent of the load for Case 4.
2. Adding midspan braces results in a dramatic increase in the buckling load capacity when only one span is loaded. This can clearly be seen by the ratio 0.796/0.193 = 4.12 indicating a more than four times increase in the buckling capacity of the two-span beam owing to the addition of midspan braces.
3. When both spans are loaded in the presence of midspan braces, the buckling load is 5.5 times compared to that in the absence of such braces.

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