Performance Analysis of Permutation Matrix Zero Cross Correlation Code for SAC-OCDMA Systems

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Abstract—SAC-OCDMA systems are picking up prominence because of their property of low Multiple Access Interference (MAI). This paper deals with zero cross-correlation (ZCC) by considering permutation matrices (PM). The properties and performance analysis of PM-ZCC code is given in this study. The obtained results shown that PM-ZCC code can achieve better BER for more number of users. In PM-ZCC code, code length is neither too short nor too long which is the good property of encoder-decoder design structure.

Index Terms—Spectral Amplitude Coding (SAC); Optical Code Division Multiple Access (OCDMA); Zero Cross-Correlation (ZCC) Code; Permutation Matrix.

I. INTRODUCTION

The key point that distinguishes OCDMA from other multiple access techniques is the use of so-called orthogonal codes to allow multiple users to utilize the same overlapping spectral range without interference. An OCDMA system interjicts from many noises such as shot noise, thermal noise, dark current and multiple access interference (MAI) from other users. Surrounded by all these noises, MAI is deliberated as the dominated source, which leads to design of code pattern by reducing MAI and phase-induced intensity noise (PIIN) rising from spontaneous emission of broadband source [11]-[5].

In Optical Spectrum Code Division Multiple Access (OSCDMA) systems, numerous codes have suggested namely OOC [6], MDW [7], MFH [8] and many more. However, these codes have found disadvantageous such as constructions are either intricate (e.g., OOC and MFH codes), the CC are not ideal or the code length is not acceptable (e.g., OOC and Prime code). If the length of the code is long this cause the drawbacks that, since either very wide band sources or very narrow filter bandwidths are needed. Due to this reason, researcher develop the ZCC codes [7], [9], [10], [11], [13]. The benefits of codes with ZCC have less noise, which effects in reducing the hardware complexity. In this paper, we give the performance analysis of PM-ZCC codes. The benefits of PM-ZCC codes are: i) Flexibility of choosing number of code weight and number of users (free cardinality); ii) zero cross-correlation.

This paper is structured as follows: The preliminaries are given in section II. The code properties PM-ZCC offered in Section III. Mathematical analysis drawn in Section IV. System description in Section V. The results and discussion given in Section VI. Finally, conclusion is given in Section VII.

II. DEFINITIONS, NOTATIONS AND RESULTS

A. Permutation Matrix

For the A permutation matrix is a square matrix attained from the same size unit matrix by a permutation of rows [12]. The examples of PM of different orders as follows:

A 2x2 PM is

\[
\begin{bmatrix}
1 & 0 \\
0 & 1 \\
\end{bmatrix}
\]

More clearly, all PM is a product of elementary row interchange matrices. In view of this idea, one can get six 3x3-PM and one of them is,

\[
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0 \\
\end{bmatrix}
\]

To continue this study, we recall the following properties of existing codes in the literature:

B. Double weight code (DW):

The DW [13] code denoted by matrix, K is the number of users and C is the minimum code length. A 2x3-DW matrix represented as

\[
DW_{M=1} = \begin{bmatrix}
0 & 1 & 1 \\
1 & 1 & 0 \\
\end{bmatrix}
\]

To support more users, the technique mentioned in the same papers (see [14],[15])

C. Modified Double Weight code (MDW)

The MDW (W>2) built by using matrix, K denotes the number of users and C is the minimum code length. The following technique adopted to increase K,
\[ Z_{M-2} = \begin{bmatrix} 0 & Z_1 \\ Z_1 & 0 \end{bmatrix} \] (2)

Very recently, Nisar [16] developed a new ZCC code by considering a type of anti-diagonal-identity- column-block matrices where \( N \) is the number of users and \( L \) (columns) represents the minimum code length. A basic new ZCC [16] for \( W = 1 \) shown in (3)

\[ N_{W=1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \] (3)

To increase the number of users and code weight, the author adopted the following mapping technique.

\[ N_{W=2} = \begin{bmatrix} 0 & 0 & 0 & 1 & : & 0 & 0 & : & 1 \\ 0 & 0 & 0 & 1 & : & 0 & 1 & : & 0 \\ 0 & 0 & 1 & 0 & : & 0 & 1 & : & 0 \\ 1 & : & 1 & 0 & : & 0 & 0 & : & 0 \end{bmatrix} \] (4)

which is the code pattern for \( W = 2 \).

III. THE PM-ZCC CODE CONSTRUCTION

A new zero cross-correlation code PM-ZCC denoted by the matrix of order \( N \times L \), where \( N \) is the number of users and \( L \) is the minimum code length. A basic PM-ZCC code for \( W = 1 \) and, \( N = 2 \) denoted by \( P_{W,N} \) is shown in (5)

\[ P_{W,2} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \] (5)

Also, we have \( P_{W,3} \) and \( P_{W,4} \) as follows:

\[ P_{W,3} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \] (6)

\[ P_{W,4} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \] (7)

From the definition of permutation matrix, it is easy to obtain one of the 5x5-permutation matrix as:

\[ P_{W,5} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \] (8)

Notice that the number of users increases with the increasing code length. To increase the code weight, we use the following matrix pattern:

For \( W = 2 \) and \( N = 4 \)

\[ P_{2,4} = \begin{bmatrix} 0 & 0 & 1 & 0 & : & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & : & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & : & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & : & 0 & 1 & 0 & 0 \end{bmatrix} \] (9)

For \( W = 3 \)

\[ P_{3,4} = \begin{bmatrix} 0 & 0 & 1 & 0 & : & 0 & 0 & 1 & 0 & : & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & : & 0 & 0 & 0 & 1 & : & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & : & 1 & 0 & 0 & 0 & : & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & : & 0 & 1 & 0 & 0 & : & 0 & 1 & 0 & 0 \end{bmatrix} \] (10)

More generally, if \( W = N \) then we have

\[ P_{n,N} = n \times N \] (11)

From the above matrix pattern, we obtained the equation of code length \( L \) as:

\[ L = N \times W \] (12)

Using (11) and (12) one can easily construct the code for \( W = 2 \) and \( N = 5 \):

\[ P_{2,5} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \] (13)

The block diagram of PMC matrix pattern given in Fig. 1.

IV. MATHEMATICAL ANALYSIS OF NEW PM-ZCC CODE

By using Gaussian approximation BER and SNR of the system are calculated to examine the performance of the system. For analysis, shot and thermal noises are considered and PIIN is ignored because of zero cross-correlation condition and no overlapping of spectra from different users.

As a result, the photodetector variance of unpolarized thermal light detection is given as [17],[18]:

\[ \text{variance} = \sigma_{\text{shot}}^2 + \sigma_{\text{thermal}}^2 \] (14)
\[
\sigma_{\text{shot}} = 2eBl
\]
\[(15)\]
\[
\sigma_{\text{thermal}} = \frac{4K_BT_n}{R_L}
\]
\[(16)\]
where \(e, B, l, K_b, T_n, R_L\) are electron charge, electrical bandwidth, average photocurrent, Boltzmann constant, receiver noise temperature and receiver load resistance respectively.

Let \(A_x(n)\) be the \(n^{th}\) element of the \(x^{th}\) PM-ZCC code sequence:

\[
\sum_{n=1}^{N} A_x(n)A_y(n) = \begin{cases} W & \text{for } x = l \text{ auto correlation (} A_x \text{)} \\ 0 & \text{for } x \neq l \text{ auto correlation (} A_x \text{)} \end{cases}
\]
\[(17)\]
where \(A_x\) and \(A_y\) are the PM-ZCC code sequences, \(N\) is a number of the user.

The BER is analyzed by considering the following assumptions [17 – 19]:

a) Every light source is ideally polarized and for the given bandwidth, its spectrum is flat. Here, \(\nu_0\) is the central optical frequency and \(\Delta \nu\) is optical source bandwidth.

b) Every single bit stream is synchronized with respect to each user.

c) The spectral width is identical for every power frequency component.

d) At the transmitter, each user possesses the same power.

From (16, 17), power spectral density (PSD) of the received output signal is given by:

\[
r(v) = \frac{P_{sr}}{\Delta \nu} \sum_{x=1}^{X} d_x \sum_{n=1}^{N} A_x(n) \text{rec}(n)
\]
\[(18)\]
where \(d_x\) is \(x^{th}\) user data bit, \(P_{sr}\) is the effective power of a broadband source at the receiver and \(\text{rec}(n)\) is the rectangle function. The \(\text{rec}(n)\) equation is given below:

\[
\text{rec}(n) = \left[ \frac{\Delta \nu}{T} \right]^n
\]
\[(19)\]

The summation of data:

\[
\sum_{x=1}^{X} d_x = d_1 + d_2 + \ldots + d_x
\]

Further,

\[
\int_0^\infty G(v)dv = \int_0^\infty \frac{P_{sr}}{\Delta \nu} \sum_{x=1}^{X} d_x \sum_{n=1}^{N} A_x(n)A_y(n) \text{rec}(n)dv
\]
\[
= \frac{P_{sr}}{\Delta \nu} \frac{\Delta \nu}{T} \sum_{x=1}^{X} d_x \sum_{n=1}^{N} A_x(n)A_y(n)
\]
\[(20)\]
\[(21)\]

\[
\text{variance}(\sigma^2) = 2eB \frac{9\text{Re}(W)}{T} + 4K_BT_n
\]
\[(22)\]
From (22) Probability of sending 1 bit at any time for each user is half

\[
\sigma^2 = eB \frac{9\text{Re}(W)}{T} + 4K_BT_n
\]
\[(23)\]

Further signal to noise ratio (SNR) is given by:

\[
\text{SNR} = \frac{\left(\frac{9\text{Re}(W)}{T}\right)^2}{eB \frac{9\text{Re}(W)}{T} + 4K_BT_n}
\]
\[(24)\]

Using Gaussian approximation, BER can be expressed as [17]-[21].

\[
\text{BER} = P_0 = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{\text{SNR}}{8}} \right)
\]
\[(25)\]

V. SYSTEM DESCRIPTION

The performance of proposed PM-ZCC code is analyzed using spectral direct detection (SDD) technique. SDD is a practical strategy, as it requires one set of the decoder in contrast to complementary subtraction technique. It additionally wipes out MAI and suppresses PIIN at the receiver, which subsequently enhances the performance of the system. [18]. The simulation is carried out using OptiSystem 13.0. A block diagram of five users PM-ZCC code having weight two (W=2) employing SAC-OCMDA using direct detection is shown in Fig. 1. The transmitter section comprises of a laser, PM-ZCC encoder followed by a power combiner. The PM-ZCC encoder is designed to using PRBS, NRZ pulse generator, MZ modulator and MUX. The Block diagram of PM-ZCC code employing SAC-OCMDA using direct detection is given in Fig. 2.

The laser power is fragment into 10 wavelengths with 0.8nm chip dispersing. Employing 5:1 power combiner modulator output signals are merged and launched into the
optical fibre. At receiver, optical power is split into 5 parts and signals are recuperated using PM-ZCC decoder which comprises of MUX, Photodiode and LPF.

VI. RESULTS & DISCUSSION

The simulation was conducted at a bit rate of 10 x 5 Gb/s for disparate ranges using single mode optical fibre. The operational wavelength (λ) is taken to be 1550nm, light source power is fixed at 0dBm and attenuation is 0.2dB/km. The dispersion and transmission losses are 16.75ps/nm/km and 1 respectively. The group velocity dispersion, third order dispersion and self-phase modulation are also used in this simulation. The receiver noise temperature (TR), electrical bandwidth and receiver load resistance (RL) are set as 300K, 80MHz and 1030 ohm respectively. The Quantum efficiency of the photodetector (PD) is taken to be 0.6. For every photodetector, the thermal noise is fixed at $1.8 \times 10^{-23}$ W/Hz. The cutoff frequency, dark current, gain and responsivity of the low-pass filter are set at 0.65×bitrate, 10nA, 3.5 and 1 A/W respectively. The performance of the proposed system is characterized by referring SNR, BER and Q-factor.

Fig. 3 represents the graph between BER and range for different numbers of users of the proposed PM-ZCC code employing SAC-OCDMA technique. From the graph, it is clear that with an increase in the number of users, BER increases. At a distance of 100 KM the BER values of five users are 7.41E-18, 6.62E-16, 9.63E-14, 9.73E-13 and 8.73E-11 with acceptable SNR value of 37.42 and Q factor of 9.49. Whereas with the same bit rate i.e. 10GB/s, if we increase the range, it can accommodate more number of users as SNR received at a distance of 160Km is 25.79 as shown in Fig. 4. Further, at a bit rate of 20GB/s the received BER for 10 users at a distance of 60 Km is 8.36E-15. So, by decreasing the distance we can increase data rate and the number of users. Fig. 5 shows a comparison of different codes (existing ZCC, MD, RD and EDW) with PM-ZCC for 30 users using similar simulation parameters as mentioned earlier. Code length is the important parameter in designing a code in Spectral Amplitude Coding OCDMA. For the given number of users and code weight, a code with minimum code length is preferable. The results show that PM-ZCC code can achieve better BER for more number of users. In PM-ZCC code, code length is neither too short nor too long which is the good property of encoder-decoder design structure. The calculated BER at a distance of 20 km for PM-ZCC, ZCC, MD, RD and EDW codes are 6.58E-46, 7.54E-36, 9.82E-36, 2.86E-33 and 2.94E-29 respectively.

![Fig. 2. Block diagram of PM-ZCC code employing SAC-OCDMA using direct detection.](image)

![Fig. 3. Plot between BER Vs Range for five users](image)

![Fig. 4. Plot between SNR Vs Range](image)
The performance of any SAC-OCDMA system strongly depends on the properties of codes. Cross-correlation is the most important property of the code. In this paper, we analyze the performance of PM-ZCC code. The results showed that the PM-ZCC code could attain better BER for more number of users. In PM-ZCC code, code length is neither too short nor too long which is the good property of encoder-decoder design structure.

REFERENCES

Fig. 5. Comparison of PM-ZCC code with other SAC-OCDMA code

VII. CONCLUDING REMARK

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